Chapter 5
Contrasts for one-way ANOVA

Contrasts for one-way ANOVA

1. What is a contrast?
   - A focused test of means
   - A weighted sum of means
   - Contrasts allow you to test your research hypothesis (as opposed to the statistical hypothesis)

   - Example: You want to investigate if a college education improves SAT scores. You obtain five groups with $n = 25$ in each group:
     - High School Seniors
     - College Seniors
       - Mathematics Majors
       - Chemistry Majors
       - English Majors
       - History Majors
     - All participants take the SAT and scores are recorded

     - The omnibus F-test examines the following hypotheses:

       $H_0 : \mu_1 = \mu_2 = \mu_3 = \mu_4 = \mu_5$

       H_1 : Not all μ_i's are equal

     - But you want to know:
       - Do college seniors score differently than high school seniors?
       - Do natural science majors score differently than humanities majors?
       - Do math majors score differently than chemistry majors?
       - Do English majors score differently than history majors?

HS Students	College Students			
	Math	Chemistry	English	History
μ_1	μ_2	μ_3	μ_4	μ_5

- Do college seniors score differently than high school seniors?

HS Students	Math	College Students Chemistry	English	History
μ_1		$\frac{\mu_2+\mu_3+\mu_4+\mu_5}{4}$		

$$H_0: \mu_1 = \frac{\mu_2+\mu_3+\mu_4+\mu_5}{4} \qquad H_1: \mu_1 \neq \frac{\mu_2+\mu_3+\mu_4+\mu_5}{4}$$

- Do natural science majors score differently than humanities majors?

HS Students	Math	College Students Chemistry	English	History
	$\frac{\mu_2+\mu_3}{2}$		$\frac{\mu_4+\mu_5}{2}$	

$$H_0: \frac{\mu_2+\mu_3}{2} = \frac{\mu_4+\mu_5}{2} \qquad H_1: \frac{\mu_2+\mu_3}{2} \neq \frac{\mu_4+\mu_5}{2}$$

- Do math majors score differently than chemistry majors?

HS Students	Math	College Students Chemistry	English	History
	μ_2	μ_3		

$$H_0: \mu_2 = \mu_3 \qquad H_1: \mu_2 \neq \mu_3$$

- Do English majors score differently than history majors?

HS Students	Math	College Students Chemistry	English	History
			μ_4	μ_5

$$H_0: \mu_4 = \mu_5 \qquad H_1: \mu_4 \neq \mu_5$$

- In general, a contrast is a set of weights that defines a specific comparison over the cell means

$$\psi = \sum_{j=1}^{a} c_i \mu_i = c_1\mu_1 + c_2\mu_2 + c_3\mu_3 + ... + c_a\mu_a$$

$$\hat{\psi} = \sum_{j=1}^{a} c_i \overline{X}_i = c_1\overline{X}_1 + c_2\overline{X}_2 + c_3\overline{X}_3 + ... + c_a\overline{X}_a$$

  - Where
    $(\mu_1, \mu_2, \mu_3, ..., \mu_a)$ are the population means for each group
    $(\overline{X}_1, \overline{X}_2, \overline{X}_3, ..., \overline{X}_a)$ are the observed means for each group
    $(c_1, c_2, c_3, ..., c_a)$ are weights/contrast coefficients
    with $\sum_{i=1}^{a} c_i = 0$

- A contrast is a linear combination of cell means

  - Do college seniors score differently than high school seniors?

    $H_0 : \mu_1 = \frac{\mu_2 + \mu_3 + \mu_4 + \mu_5}{4}$ or $H_0 : \mu_1 - \frac{\mu_2 + \mu_3 + \mu_4 + \mu_5}{4} = 0$

    $\psi_1 = \mu_1 - \frac{1}{4}\mu_2 - \frac{1}{4}\mu_3 - \frac{1}{4}\mu_4 - \frac{1}{4}\mu_5$ $\quad c = \left(1, -\frac{1}{4}, -\frac{1}{4}, -\frac{1}{4}, -\frac{1}{4}\right)$

  - Do natural science majors score differently than humanities majors?

    $H_0 : \frac{\mu_2 + \mu_3}{2} = \frac{\mu_4 + \mu_5}{2}$ or $H_0 : \frac{\mu_2 + \mu_3}{2} - \frac{\mu_4 + \mu_5}{2} = 0$

    $\psi_2 = \frac{1}{2}\mu_2 + \frac{1}{2}\mu_3 - \frac{1}{2}\mu_4 - \frac{1}{2}\mu_5$ $\quad c = \left(0, \frac{1}{2}, \frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}\right)$

  - Do math majors score differently than chemistry majors?

    $H_0 : \mu_2 = \mu_3$ or $H_0 : \mu_2 - \mu_3 = 0$

    $\psi_3 = \mu_2 - \mu_3$ $\quad c = (0, 1, -1, 0, 0)$

2. Types of Contrasts

- Pairwise contrasts
  - Comparisons between two cell means
  - Contrast is of the form $c_i = 1$ and $c_{i'} = -1$ for some *i* and i'
  - If you have *a* groups then there are $\frac{a(a-1)}{2}$ possible pairwise contrasts
  - Examples:
    - Do math majors score differently than chemistry majors?

      $\psi_3 = \mu_2 - \mu_3 \qquad c = (0,1,-1,0,0)$
    - Do English majors score differently than history majors?

      $\psi_4 = \mu_4 - \mu_5 \qquad c = (0,0,0,1,-1)$

  - When there are two groups ($a = 2$), then the two independent samples t-test is equivalent to the $c = (1,-1)$ contrast on the two means.

- Complex contrasts
  - A contrast between more than two means
  - There are an infinite number of contrasts you can perform for any design

  - Do college seniors score differently than high school seniors?

    $$\psi_1 = \mu_1 - \frac{1}{4}\mu_2 - \frac{1}{4}\mu_3 - \frac{1}{4}\mu_4 - \frac{1}{4}\mu_5 \qquad c = \left(1,-\frac{1}{4},-\frac{1}{4},-\frac{1}{4},-\frac{1}{4}\right)$$

  - Do natural science majors score differently than humanities majors?

    $$\psi_2 = \frac{1}{2}\mu_2 + \frac{1}{2}\mu_3 - \frac{1}{2}\mu_4 - \frac{1}{2}\mu_5 \qquad c = \left(0,\frac{1}{2},\frac{1}{2},-\frac{1}{2},-\frac{1}{2}\right)$$

  - So long as the coefficients sum to zero, you can make any comparison:

    $$\psi_k = .01\mu_1 - .08\mu_2 - .98\mu_3 + .58\mu_4 + .47\mu_5 \qquad c = (.01,-.08,-.98,.58,.47)$$

    - But remember you have to be able to interpret the result!

- Orthogonal contrasts
  - Sometimes called non-redundant contrasts
  - Orthogonality may be best understood through a counter-example

  - Suppose you want to test three contrasts:
    - Do math majors score differently than high school seniors?

      $\psi_1 = \mu_2 - \mu_1$ $\qquad c = (-1,1,0,0,0)$
    - Do chemistry majors score differently than high school seniors?

      $\psi_2 = \mu_3 - \mu_1$ $\qquad c = (-1,0,1,0,0)$
    - Do math majors score differently than chemistry majors?

      $\psi_3 = \mu_2 - \mu_3$ $\qquad c = (0,1,-1,0,0)$

  - But we notice that

$$\psi_1 = \mu_2 - \mu_1 = \mu_2(-\mu_3 + \mu_3) - \mu_1 = (\mu_2 - \mu_3) + (\mu_3 - \mu_1) = \psi_3 + \psi_2$$

    - If I know ψ_2 and ψ_3 then I can determine the value of ψ_1
    - ψ_1, ψ_2, and ψ_3 are redundant or <u>non-orthogonal</u> contrasts

  - Orthogonality defined:
    - A set of contrasts is orthogonal if they are independent of each other (or if knowing the value of one contrast in no way provides any information about the other contrast)

    - If a set of contrasts are orthogonal then the contrast coefficients are not correlated with each other

    - Two contrasts are orthogonal if the angle between them in a-space is a right angle

    - Two contrasts are orthogonal if for equal *n*

$$\psi_1 = (a_1, a_2, a_3, ..., a_a)$$
$$\psi_2 = (b_1, b_2, b_3, ..., b_a)$$

$$\sum_{j=1}^{a} a_i b_i = 0 \text{ or } \qquad a_1 b_1 + a_2 b_2 + ... + a_a b_a = 0$$

- Two contrasts are orthogonal if for unequal *n*

$$\psi_1 = (a_1, a_2, a_3, ..., a_a)$$
$$\psi_2 = (b_1, b_2, b_3, ..., b_a)$$

$$\sum_{j=1}^{a} \frac{a_i b_i}{n_i} = 0 \text{ or } \qquad \frac{a_1 b_1}{n_1} + \frac{a_2 b_2}{n_2} + ... + \frac{a_a b_a}{n_a} = 0$$

- Examples of Orthogonality (assuming equal n)

  - Set #1: $c_1 = (1,0,-1)$ and $c_2 = \left(\frac{1}{2}, -1, \frac{1}{2}\right)$

$$\sum_{j=1}^{a} c_{1i} c_{2i} = \left(1 * \frac{1}{2}\right) + (0 * -1) + \left(-1 * \frac{1}{2}\right)$$
$$= \frac{1}{2} + 0 - \frac{1}{2} = 0$$

c_1 and c_2 are orthogonal

$c_1 \perp c_2$

  - Set #2: $c_3 = (0,1,-1)$ and $c_4 = \left(-1, \frac{1}{2}, \frac{1}{2}\right)$

$$\sum_{j=1}^{a} c_{3i} c_{4i} = (0 * -1) + \left(1 * \frac{1}{2}\right) + \left(-1 * \frac{1}{2}\right)$$
$$= 0 + \frac{1}{2} - \frac{1}{2} = 0$$

c_3 and c_4 are orthogonal

$c_3 \perp c_4$

  - Set #3: $c_5 = (1,-1,0)$ and $c_6 = (1,0,-1)$

$$\sum_{j=1}^{a} c_{5i} c_{6i} = (1 * 1) + (-1 * 0) + (0 * -1)$$
$$= 1 + 0 + 0 = 1$$

c_5 and c_6 are NOT orthogonal

- A set of contrasts is orthogonal if each contrast is orthogonal to all other contrasts in the set

	You can check that:
$c_1 = (1,-1,0,0)$	$c_1 \perp c_2$
$c_2 = (1,1,-2,0)$	$c_2 \perp c_3$
$c_3 = (1,1,1,-3)$	$c_1 \perp c_3$

- If you have a groups, then there are a-1 possible orthogonal contrasts
  - We lose one contrast for the grand mean (the unit contrast)
  - Having the contrasts sum to zero assures that they will be orthogonal to the unit contrast
  - If you have more than a-1 contrasts, then the contrasts are redundant and you can write at least one contrast as a linear combination of the other contrasts

  - Example: For a=3, we can find only 2 orthogonal contrasts. Any other contrasts are redundant.

$\psi_1 = \mu_1 - \mu_2$	$\psi_1 \perp \psi_2$
$\psi_2 = \frac{1}{2}\mu_1 + \frac{1}{2}\mu_2 - \mu_3$	ψ_1 is not orthogonal to ψ_3
$\psi_3 = -\mu_1 + \frac{4}{5}\mu_2 + \frac{1}{5}\mu_3$	ψ_2 is not orthogonal to ψ_3

  We can write ψ_3 in terms of ψ_1 and ψ_2

$$\begin{aligned}\psi_3 &= -\frac{9}{10}\psi_1 - \frac{1}{5}\psi_2 \\ &= -\frac{9}{10}(\mu_1 - \mu_2) - \frac{1}{5}\left(\frac{1}{2}\mu_1 + \frac{1}{2}\mu_2 - \mu_3\right) \\ &= \left(-\frac{9}{10}\mu_1 + \frac{9}{10}\mu_2\right) + \left(-\frac{1}{10}\mu_1 - \frac{1}{10}\mu_2 + \frac{1}{5}\mu_3\right) \\ &= -\mu_1 + \frac{4}{5}\mu_2 + \frac{1}{5}\mu_3\end{aligned}$$

  - In general, you will not need to show how a contrast may be calculated from a set of orthogonal contrasts. It is sufficient to know that if you have more than a-1 contrasts, there must be at least one contrast you can write as a linear combination of the other contrasts.

- There is nothing wrong with testing non-orthogonal contrasts, as long as you are aware that they are redundant.
- For example, you may want to examine all possible pairwise contrasts. These contrasts are not orthogonal, but they may be relevant to your research hypothesis.

- Nice properties of orthogonal contrasts:
  - We will learn to compute Sums of Squares associated with each contrast (SSC_i)

  - For a set of a-1 orthogonal contrasts
    - Each contrast has one degree of freedom

      $$SSB = SSC_1 + SSC_2 + SSC_3 + \ldots + SSC_{a-1}$$

    - In other words, a set of a-1 orthogonal contrasts partitions the SSB

    - Recall that for the omnibus ANOVA, the $df_{bet} = a - 1$. The omnibus test combines the results of these a-1 contrasts and reports them in one lump test
    - Any set of a-1 orthogonal contrasts will yield the identical result as the omnibus test

3. Significance tests of a single contrast
   - Recall the general form of a t-test:

$$t \sim \frac{\text{estimate of population parameter}}{\text{estimated standard error}}$$

$$t \sim \frac{\hat{\psi}}{\text{standard error}(\hat{\psi})}$$

   - To compute a significance test for a single contrast, we need:
     - An estimate of the value of the contrast
     - An estimate of the standard error of the contrast
       (The standard deviation of the sampling distribution of the contrast)

     - Value of a contrast:

$$\psi = \sum_{j=1}^{a} c_i \mu_i = c_1\mu_1 + c_2\mu_2 + c_3\mu_3 + \ldots + c_a\mu_a$$

$$\hat{\psi} = \sum_{j=1}^{a} c_i \overline{X}_i = c_1\overline{X}_1 + c_2\overline{X}_2 + c_3\overline{X}_3 + \ldots + c_a\overline{X}_a$$

$\hat{\psi}$ is an unbiased estimate of the true population value of ψ

     - Standard error of a contrast
       - Recall that standard error is the standard deviation of the sampling distribution

       - The standard error for the two independent samples t-test:

$$Std\ Error = s_{pooled}\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}$$

       - The standard error of a contrast has a similar form:

$$Std\ Error(\hat{\psi}) = \sqrt{MSW\sum_{i=1}^{a}\frac{c_i^2}{n_i}}$$

Where c_i^2 is the squared weight for each group
n_i is the sample size for each group
MSW is MSW from the omnibus ANOVA

- Constructing a significance test

$$t \sim \frac{\text{estimate of population parameter}}{\text{estimated standard error}}$$

$$t \sim \frac{\hat{\psi}}{\text{standard error}(\hat{\psi})}$$

- Now we can insert the parts of the t-test into the equation:

$$t_{observed} = \frac{\sum c_i \overline{X}_i}{\sqrt{MSW \sum \frac{c_i^2}{n_i}}}$$

with degrees of freedom $= df_w = N - a$

- To determine the level of significance, you can:
  - Look up t_{crit} for $df = N\text{-}a$ and the appropriate α
  - Or preferably, compare p_{obs} with p_{crit}

- Note that because the test for a contrast is calculated using the t-distribution, you can use either a one-tailed or two-tailed test of significance. As previously mentioned, you typically want to report the two-tailed test of the contrast.

- Example #1: a=2 and $c = (1,-1)$

$$\hat{\psi} = (1)\overline{X}_1 + (-1)\overline{X}_2 = \overline{X}_1 - \overline{X}_2$$

$$Std\ error\ (\hat{\psi}) = \sqrt{MSW\left(\frac{1^2}{n_1} + \frac{(-1)^2}{n_2}\right)} = \sqrt{MSW}\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}$$

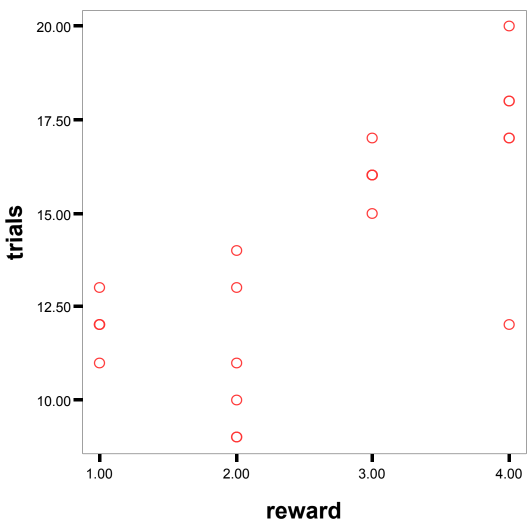
$$t = \frac{\hat{\psi}}{StdError(\hat{\psi})} = \frac{\overline{X}_1 - \overline{X}_2}{\sqrt{MSW\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}}$$

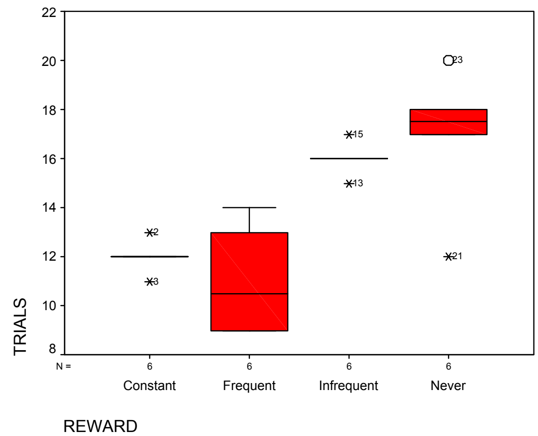
  - But we know that for two groups, $\sqrt{MSW} = s_{pooled}$
  - Thus, the two independent samples t-test is identical to a $c = (1,-1)$ contrast on the two means

- Example #2: A study of the effects of reward on learning in children
  DV = Number of trials to learn a puzzle

Level of Reward			
Constant (100%)	Frequent (66%)	Infrequent (33%)	Never (0%)
12	9	15	17
13	10	16	18
11	9	17	12
12	13	16	18
12	14	16	20
12	**11**	**16**	**17**

H1: Constant reward will produce faster learning than the average of the other conditions
H2: Frequent reward will produce faster learning than the average of infrequent or no reward
H3: Infrequent reward will produce faster learning than no reward





- Step 1: Convert the research hypothesis into a contrast of means

  - H1: Constant reward will produce faster learning than the average of the other conditions

$$H_0 : \mu_1 = \frac{\mu_2 + \mu_3 + \mu_4}{3} \qquad H_1 : \mu_1 \neq \frac{\mu_2 + \mu_3 + \mu_4}{3}$$

$$\psi_1 = \mu_1 - \frac{1}{3}\mu_2 - \frac{1}{3}\mu_3 - \frac{1}{3}\mu_4 \qquad c_1 = \left(1, -\frac{1}{3}, -\frac{1}{3}, -\frac{1}{3}\right)$$

  - H2: Frequent reward will produce faster learning than the average of infrequent or no reward

$$H_0 : \mu_2 = \frac{\mu_3 + \mu_4}{2} \qquad H_1 : \mu_2 \neq \frac{\mu_3 + \mu_4}{2}$$

$$\psi_2 = \mu_2 - \frac{1}{2}\mu_3 - \frac{1}{2}\mu_4 \qquad c_2 = \left(0, 1, -\frac{1}{2}, -\frac{1}{2}\right)$$

  - H3: Infrequent reward will produce faster learning than no reward

$$H_0 : \mu_3 = \mu_4 \qquad H_1 : \mu_3 \neq \mu_4$$

$$\psi_3 = \mu_3 - \mu_4 \qquad c_3 = (0, 0, 1, -1)$$

- Step 2: Determine if the contrasts of interest are orthogonal

$$c_1 \& c_2: \quad \sum_{j=1}^{4} c_{1i}c_{2i} = (1*0) + \left(-\frac{1}{3}*1\right) + \left(-\frac{1}{3}*-\frac{1}{2}\right) + \left(-\frac{1}{3}*-\frac{1}{2}\right) = 0 \qquad c_1 \perp c_2$$

$$c_1 \& c_3: \quad \sum_{j=1}^{4} c_{1i}c_{3i} = (1*0) + \left(-\frac{1}{3}*0\right) + \left(-\frac{1}{3}*1\right) + \left(-\frac{1}{3}*-1\right) = 0 \qquad c_1 \perp c_3$$

$$c_2 \& c_3: \quad \sum_{j=1}^{4} c_{2i}c_{3i} = (0*0) + (1*0) + \left(-\frac{1}{2}*1\right) + \left(-\frac{1}{2}*-1\right) = 0 \qquad c_2 \perp c_3$$

- Step 3: Compute values for each contrast

$$\hat{\psi}_1 = \overline{X}_1 - \frac{1}{3}\overline{X}_2 - \frac{1}{3}\overline{X}_3 - \frac{1}{3}\overline{X}_4 \qquad \hat{\psi}_2 = \overline{X}_2 - \frac{1}{2}\overline{X}_3 - \frac{1}{2}\overline{X}_4$$

$$= 12 - \frac{1}{3}(11) - \frac{1}{3}(16) - \frac{1}{3}(17) \qquad = (11) - \frac{1}{2}(16) - \frac{1}{2}(17)$$

$$= -2.6667 \qquad = -5.5$$

$$\hat{\psi}_3 = \overline{X}_3 - \overline{X}_4$$

$$= (16) - (17)$$

$$= -1$$

- Step 4: Conduct omnibus ANOVA to obtain *MSW*

ANOVA

Source of Variation	*SS*	*df*	*MS*	*F*	*P-value*	*F crit*
Between Groups	130	3	43.33333	11.1828	0.000333	3.238867
Within Groups	62	16	3.875			
Total	192	19				

  - Note: we do not care about the results of this test. We only want to calculate *MSW*

- Step 5: Compute standard error for each contrast

$$Std\ Error(\hat{\psi}) = \sqrt{MSW\sum_{i=1}^{a}\frac{c_i^2}{n_i}}$$

$$\text{Std err }(\hat{\psi}_1) = \sqrt{3.875\left(\frac{1^2}{5}+\frac{\left(-\frac{1}{3}\right)^2}{5}+\frac{\left(-\frac{1}{3}\right)^2}{5}+\frac{\left(-\frac{1}{3}\right)^2}{5}\right)} = \sqrt{3.875*.2667} = 1.0165$$

$$\text{Std err }(\hat{\psi}_2) = \sqrt{3.875\left(0+\frac{(1)^2}{5}+\frac{\left(-\frac{1}{2}\right)^2}{5}+\frac{\left(-\frac{1}{2}\right)^2}{5}\right)} = \sqrt{3.875*.30} = 1.0782$$

$$\text{Std err }(\hat{\psi}_3) = \sqrt{3.875\left(0+0+\frac{(1)^2}{5}+\frac{(-1)^2}{5}\right)} = \sqrt{3.875*.40} = 1.2450$$

- Step 6: Compute observed t or F statistic for each contrast

$$t_{observed} = \frac{\sum c_i \overline{X}_i}{\sqrt{MSW\sum\frac{c_i^2}{n_i}}}$$

$$\hat{\psi}_1:\ t_{observed} = \frac{-2.6667}{1.0165} = 2.6237 \qquad \hat{\psi}_2:\ t_{observed} = \frac{-5.5}{1.0782} = -5.1011$$

$$\hat{\psi}_3:\ t_{observed} = \frac{-1}{1.2450} = -0.8032$$

- Step 7: Determine statistical significance
    - Method 1 (The table method): Find t_{crit} for *df* = *N-a* and $\alpha = \frac{.05}{2} = .025$

      $t_{crit}(16)_{\alpha=.025} = 2.12$

      Compare t_{crit} to t_{obs}

      if $|t_{observed}| < |t_{critical}|$ then retain H_0
      if $|t_{observed}| \geq |t_{critical}|$ then reject H_0

      We reject the null hypothesis for $\hat{\psi}_1$ and $\hat{\psi}_2$

        - Constant reward produced faster learning than the average of the other conditions
        - Frequent reward produced faster learning than the average of infrequent or no reward

    - Method 2: (The exact method): Find p_{obs} for *df* = *N-a* and $\alpha = \frac{.05}{2} = .025$ for each t_{obs} and Then compare p_{obs} to $p_{crit} = .05$

      ψ_1 : $t_{observed}(16) = 2.62, p = .02$
      ψ_2 : $t_{observed}(16) = -5.10, p < .01$
      ψ_3 : $t_{observed}(16) = -0.80, p = .43$

- Alternatively, you can perform an F-test to evaluate contrasts.
    - We know that $t^2 = F$

$$t_{observed} = \frac{\sum c_i \overline{X}_i}{\sqrt{MSW \sum \frac{c_i^2}{n_i}}} \qquad F_{observed} = \frac{\hat{\psi}^2}{MSW \sum \frac{c_i^2}{n_i}}$$

df = *N-a* *df* = *(1, N-a)*

    - You will obtain the exact same results with t-tests or F-tests.

- Confidence intervals for contrasts
  - In general, the formula for a confidence interval is

$$estimate \pm \left(t_{critical} * standard\ error\right)$$

  - For a contrast, the formula for a confidence interval is

$$estimate \pm \left(t_{critical} * standard\ error\right)$$

$$\hat{\psi} \pm \left(t_{critical}(dfw) \ * \sqrt{MSW \sum \frac{c_i^2}{n_i}} \right)$$

    - In the learning example, $t_{crit}(16)_{\alpha=.025} = 2.12$

$$\psi_1: \ -2.667 \pm (2.12 \ * 1.0165) \qquad (-4.82, -0.51)$$
$$\psi_2: \ -5.5 \pm (2.12 \ * 1.0782) \qquad (-7.79, -3.21)$$
$$\psi_3: \ -1.0 \pm (2.12 \ * 1.2450) \qquad (-3.64, 1.64)$$

- Using SPSS to evaluate contrasts
  - If you use ONEWAY, you can directly enter the contrast coefficients to obtain the desired contrast.

```
ONEWAY trials BY reward
 /STAT desc
 /CONTRAST = 1, -.333, -.333, -.333
 /CONTRAST = 0, 1, -.5, -.5
 /CONTRAST = 0, 0, 1, -1.
```

Descriptives

TRIALS

	N	Mean	Std. Deviation	Std. Error	95% Confidence Interval for Mean	
					Lower Bound	Upper Bound
Constant	5	12.0000	.70711	.31623	11.1220	12.8780
Frequent	5	11.0000	2.34521	1.04881	8.0880	13.9120
Infrequent	5	16.0000	.70711	.31623	15.1220	16.8780
Never	5	17.0000	3.00000	1.34164	13.2750	20.7250
Total	20	14.0000	3.17888	.71082	12.5122	15.4878

Contrast Coefficients

	REWARD			
Contrast	Constant	Frequent	Infrequent	Never
1	1	-.333	-.333	-.333
2	0	1	-.5	-.5
3	0	0	1	-1

Contrast Tests

		Contrast	Value of Contrast	Std. Error	t	df	Sig. (2-tailed)
TRIALS	Assume equal variances	1	-2.6520	1.01628	-2.610	16	.019
		2	-5.5000	1.07819	-5.101	16	.000
		3	-1.0000	1.24499	-.803	16	.434

- Multiplying the coefficients by a constant will not change the results of the significance test on that contrast.
  - If you multiply the values of a contrast by any constant (positive or negative), you will obtain the identical test statistic and p-value in your analysis.
  - The value of the contrast, the standard error, and the size of the CIs will shrink or expand by a factor of the constant used, but key features (i.e., p-values and whether or not the CIs overlap) remain the same.

- Let's examine Hypothesis 1 using three different sets of contrast coefficients:

```
ONEWAY  trials BY reward
  /STAT desc
  /CONTRAST = 1, -.333, -.333, -.333
  /CONTRAST = 3, -1, -1, -1
  /CONTRAST = -6, 2, 2, 2.
```

Contrast Coefficients

	REWARD			
Contrast	Constant	Frequent	Infrequent	Never
1	1	-.333	-.333	-.333
2	3	-1	-1	-1
3	-6	2	2	2

Contrast Tests

		Contrast	Value of Contrast	Std. Error	t	df	Sig. (2-tailed)
TRIALS	Assume equal variances	1	-2.6520	1.01628	-2.610	16	.019
		2	-8.0000	3.04959	-2.623	16	.018
		3	16.0000	6.09918	2.623	16	.018

  - You get more precise values if you enter the exact contrast coefficients into SPSS, so try to avoid rounding decimal places. Instead, multiply the coefficients by a constant so that all coefficients are whole numbers.
  - In this case, the tests for contrasts 2 and 3 are exact. The test for contrast 1 is slightly off due to rounding.

- Sums of Squares of a contrast
  - As previously mentioned, a set of *a-1* orthogonal contrasts will perfectly partition the *SSB*:

    $SSB = SS\hat{\psi}_1 + SS\hat{\psi}_2 + SS\hat{\psi}_3$

  - To compute the Sums of Squares of a contrast:

$$SS\hat{\psi} = \frac{\hat{\psi}^2}{\sum \frac{c_i^2}{n_i}}$$

$$SS\hat{\psi}_1 = \frac{(-8)^2}{\frac{3^2}{5} + \frac{(-1)^2}{5} + \frac{(-1)^2}{5} + \frac{(-1)^2}{5}} = \frac{64}{2.4} = 26.67$$

$$SS\hat{\psi}_2 = \frac{(-5.5)^2}{0 + \frac{(1)^2}{5} + \frac{\left(-\frac{1}{2}\right)^2}{5} + \frac{\left(-\frac{1}{2}\right)^2}{5}} = \frac{30.25}{.3} = 100.83$$

$$SS\hat{\psi}_3 = \frac{(-1)^2}{0 + 0 + \frac{(1)^2}{5} + \frac{(-1)^2}{5}} = \frac{1.0}{.4} = 2.5$$

$$SS\hat{\psi}_1 + SS\hat{\psi}_2 + SS\hat{\psi}_3 = 26.67 + 100.83 + 2.5 = 130$$

$$SSB = 130$$

ANOVA						
Source of Variation	*SS*	*df*	*MS*	*F*	*P-value*	*F crit*
Between Groups	130	3	43.33333	11.1828	0.000333	3.238867
Within Groups	62	16	3.875			
Total	192	19				

- Once we have calculated the *SSC*, then we can compute an F-test directly:

$$F(1, dfw) = \frac{SSC/dfc}{SSW/dfw} = \frac{SSC}{MSW}$$

$$\hat{\psi}_1 : F_{observed} = \frac{26.67}{3.875} = 6.88 \qquad \hat{\psi}_2 : F_{observed} = \frac{100.83}{3.875} = 26.021$$

$$\hat{\psi}_3 : F_{observed} = \frac{2.50}{3.875} = 0.645$$

- ANOVA table for contrasts

ANOVA					
Source of Variation	*SS*	*df*	*MS*	*F*	*P-value*
Between Groups	130	3	43.33333	11.1828	0.000333
$\hat{\psi}_1$	26.67	1	26.37	6.8817	0.018446
$\hat{\psi}_2$	100.83	1	100.83	26.021	0.000107
$\hat{\psi}_3$	2.50	1	2.50	0.645	0.433675
Within Groups	62	16	3.875		
Total	192	19			

- In this ANOVA table, we show that *SSC* partitions *SSB*.
- But this relationship only holds for sets of orthogonal contrasts
- In general, you should only construct an ANOVA table for a set of *a-1* orthogonal contrasts

- Note: We will shortly see they you can either perform the omnibus test OR tests of orthogonal contrasts, but not both. Nevertheless, this ANOVA table nicely displays the SS partition.

4. Brand name contrasts easily obtained from SPSS
   - Difference contrasts
   - Helmert contrasts
   - Simple contrasts
   - Repeated contrasts
   - Polynomial contrasts (to be covered later)

- <u>Difference contrasts</u>: Each level of a factor is compared to the mean of the previous levels (These are orthogonal with equal n)

c_1	c_2	c_3	c_4
-1	1	0	0
-1	-1	2	0
-1	-1	-1	3

Contrast Coefficients (L' Matrix)

	REWARD Difference Contrast		
Parameter	Level 2 vs. Level 1	Level 3 vs. Previous	Level 4 vs. Previous
Intercept	.000	.000	.000
[REWARD=1.00]	-1.000	-.500	-.333
[REWARD=2.00]	1.000	-.500	-.333
[REWARD=3.00]	.000	1.000	-.333
[REWARD=4.00]	.000	.000	1.000

The default display of this matrix is the transpose of the corresponding L matrix.

```
UNIANOVA  trials  BY reward
  /CONTRAST (reward)=difference
  /PRINT = test(lmatrix)
```

- Helmert contrasts: Each level of a factor is compared to the mean of subsequent levels (These are orthogonal with equal n).
    - The researcher's original hypotheses for the learning data are Helmert contrasts.

c_1	c_2	c_3	c_4
3	-1	-1	-1
0	2	-1	-1
0	0	1	-1

Contrast Coefficients (L' Matrix)

Parameter	REWARD Helmert Contrast		
	Level 1 vs. Later	Level 2 vs. Later	Level 3 vs. Level 4
Intercept	.000	.000	.000
[REWARD=1.00]	1.000	.000	.000
[REWARD=2.00]	-.333	1.000	.000
[REWARD=3.00]	-.333	-.500	1.000
[REWARD=4.00]	-.333	-.500	-1.000

The default display of this matrix is the transpose of the corresponding L matrix.

```
UNIANOVA  trials  BY reward
  /CONTRAST (reward)=helmert
  /PRINT = test(lmatrix)
```

- Simple contrasts: Each level of a factor is compared to the last level (These contrasts are not orthogonal).

c_1	c_2	c_3	c_4
1	0	0	-1
0	1	0	-1
0	0	1	-1

Contrast Coefficients (L' Matrix)

Parameter	REWARD Simple Contrast[a]		
	Level 1 vs. Level 4	Level 2 vs. Level 4	Level 3 vs. Level 4
Intercept	0	0	0
[REWARD=1.00]	1	0	0
[REWARD=2.00]	0	1	0
[REWARD=3.00]	0	0	1
[REWARD=4.00]	-1	-1	-1

The default display of this matrix is the transpose of the corresponding L matrix.

a. Reference category = 4

```
UNIANOVA  trials  BY reward
  /CONTRAST (reward)=simple
  /PRINT = test(lmatrix).
```


- Repeated contrasts: Each level of a factor is compared to the previous level (These contrasts are not orthogonal).

c_1	c_2	c_3	c_4
1	-1	0	0
0	1	-1	0
0	0	1	-1

Contrast Coefficients (L' Matrix)

Parameter	REWARD Repeated Contrast		
	Level 1 vs. Level 2	Level 2 vs. Level 3	Level 3 vs. Level 4
Intercept	0	0	0
[REWARD=1.00]	1	0	0
[REWARD=2.00]	-1	1	0
[REWARD=3.00]	0	-1	1
[REWARD=4.00]	0	0	-1

The default display of this matrix is the transpose of the corresponding L matrix.

```
UNIANOVA  trials  BY reward
  /CONTRAST (reward)=repeated
  /PRINT = test(lmatrix).
```

5. Relationships between the omnibus F and contrasts (for equal *n* designs)

- Relationship #1: The omnibus F test is equal to the average t^2s from all possible pairwise contrasts.
  - Consequence: If the omnibus *F* test is significant, then at least one pairwise contrast is significant.

- Relationship #2: If you take the average F (or t^2s) from a set of a-1 orthogonal contrasts, the result will equal the omnibus F!

  - A mini-proof:

$$\text{For } \hat{\psi}_1: \; F_1 = \frac{SS\hat{\psi}_1 / df\hat{\psi}_1}{SSW / dfw} = \frac{SS\hat{\psi}_1}{MSW} \; . \; . \; . \; . \text{For } \hat{\psi}_{a-1}: \; F_{a-1} = \frac{SS\hat{\psi}_{a-1} / df\hat{\psi}_{a-1}}{SSW / dfw} = \frac{SS\hat{\psi}_{a-1}}{MSW}$$

$$\overline{F} = \frac{F_1 + F_2 + ... + F_{a-1}}{a-1} = \frac{\left(\frac{SS\hat{\psi}_1}{MSW} + \frac{SS\hat{\psi}_2}{MSW} + ... + \frac{SS\hat{\psi}_{a-1}}{MSW}\right)}{a-1}$$

$$= \frac{\left(\frac{SS\hat{\psi}_1 + SS\hat{\psi}_2 + ... + SS\hat{\psi}_{a-1}}{MSW}\right)}{a-1}$$

$$= \frac{\left(\frac{SSB}{MSW}\right)}{a-1} = \frac{\left(\frac{SSB}{a-1}\right)}{MSW} = \frac{MSB}{MSW} = F_{omnibus}$$

  - Consequence: If the omnibus F test is significant, then at least one contrast is significant.

- In the learning example:

ANOVA					
Source of Variation	*SS*	*df*	*MS*	*F*	*P-value*
Between Groups	130	3	43.33333	11.1828	0.000333
$\hat{\psi}_1$	26.37	1	26.37	6.8052	0.019003
$\hat{\psi}_2$	100.83	1	100.83	26.021	0.000107
$\hat{\psi}_3$	2.50	1	2.50	0.645	0.433675
Within Groups	62	16	3.875		
Total	192	19			

- Average F from the set of orthogonal contrasts:

$$\frac{6.8052 + 26.021 + .645}{3} = 11.16$$ (Difference is due to rounding error)

- Is it possible to have a significant contrast, but have a non-significant omnibus F?
  YES!

  - Let's consider an example:

IV			
Level 1	Level 2	Level 3	Level 4
1	2	2	4
2	3	3	5
3	4	4	6
4	5	5	7
5	6	6	8
3	**4**	**4**	**6**

```
ONEWAY  dv BY iv
 /CONTRAST= -1 -1 -1 3.
```

ANOVA

DV

	Sum of Squares	df	Mean Square	F	Sig.
Between Groups	23.750	3	7.917	3.167	.053
Within Groups	40.000	16	2.500		
Total	63.750	19			

- Omnibus F-test is not significant

Contrast Tests

		Contrast	Value of Contrast	Std. Error	t	df	Sig. (2-tailed)
DV	Assume equal variances	1	7.0000	2.44949	2.858	16	.011

- The contrast comparing Group 4 to the average of the other groups is significant

- Suppose none of the pairwise contrasts are significant. Is it possible to have a significant contrast?

  YES!

  - If none of the pairwise contrasts are significant, then the omnibus F test will not be significant. But you may still find a contrast that is significant!

IV			
Level 1	Level 2	Level 3	Level 4
0	2	2	1
1	3	3	2
2	4	4	3
3	5	5	4
4	6	6	5
2	**4**	**4**	**3**


```
ONEWAY dv BY iv
  /CONTRAST= 1 -1 0 0
  /CONTRAST= 1 0 -1 0
  /CONTRAST= 1 0 0 -1
  /CONTRAST= 0 1 -1 0
  /CONTRAST= 0 1 0 -1
  /CONTRAST= 0 0 -1 1.
```

- None of the pairwise contrasts are significant:

Contrast Tests

	Contrast	Value of Contrast	Std. Error	t	df	Sig. (2-tailed)
DV	1	-2.0000	1.00000	-2.000	16	.063
	2	-2.0000	1.00000	-2.000	16	.063
	3	-1.0000	1.00000	-1.000	16	.332
	4	.0000	1.00000	.000	16	1.000
	5	1.0000	1.00000	1.000	16	.332
	6	-1.0000	1.00000	-1.000	16	.332

- So we know that the omnibus F-test is not significant:

ANOVA

DV

	Sum of Squares	df	Mean Square	F	Sig.
Between Groups	13.750	3	4.583	1.833	.182
Within Groups	40.000	16	2.500		
Total	53.750	19			

- But it is still possible to find a significant contrast:

```
ONEWAY  dv BY iv
 /CONTRAST= 1 -1 -1 1.
```

Contrast Tests

	Contrast	Value of Contrast	Std. Error	t	df	Sig. (2-tailed)
DV	1	-3.0000	1.41421	-2.121	16	.050

- To reiterate:

    - A significant omnibus F-test ⇒ There will be at least 1 significant contrast

    - A significant contrast DOES NOT IMPLY a significant omnibus F-test

    - A non significant omnibus F-test DOES NOT IMPLY all contrasts will be non-significant

6. Robust tests for a single contrast

- The assumptions for contrasts are the same as those for ANOVA
    - Independent samples
    - Within each group, participants are independent and randomly selected
    - Equal population variances in each group
    - Each group is drawn from a normal population

- Tests of contrasts are not robust to heterogeneity of variances, even with equal n

- We can use our standard ANOVA techniques to test these assumptions. Presumably, by the time you are testing contrasts, you have already identified troublesome aspects about your data. But once you have identified the problems what can you do?
    - In general, the same "fixes" for ANOVA work for contrasts
    - Transformations can be used for non-normality or heterogeneous variances
    - A sensitivity analysis can be used to investigate the impact of outliers

- There are two additional tools we did not use for ANOVA
    - Use a contrast-specific variance so that we do not assume equality of variances in all groups
    - Try a pairwise rank-based alternative

- Use a contrast-specific variance
  - In the standard hypothesis test of a contrast, the denominator uses the *MSW*, a pooled variance estimate

$$t_{observed} = \frac{\sum c_i \overline{X}_i}{\sqrt{MSW \sum \frac{c_i^2}{n_i}}}$$

  - What we would like to do is compute a new standard error of $\hat{\psi}$ that does not rely on *MSW*

  - The details are messy but fortunately you do not have to do the dirty work; SPSS automatically prints out tests of contrasts with unequal variances.
    - When $a = 2$, this test reduces exactly to the Welch's separate variance two-sample t-test

  - Let's return to the learning example and pretend that we found heterogeneous variance. Thus, to test our original hypotheses in the data (see p 5-12), we need to use the modified test for contrast:

```
ONEWAY  trials BY reward
  /CONTRAST = 3, -1, -1, -1
  /CONTRAST = 0, 1, -.5, -.5
  /CONTRAST = 0, 0, 1, -1.
```

Contrast Tests

		Contrast	Value of Contrast	Std. Error	t	df	Sig. (2-tailed)
TRIALS	Assume equal variances	1	-8.0000	3.04959	-2.623	16	.018
		2	-5.5000	1.07819	-5.101	16	.000
		3	-1.0000	1.24499	-.803	16	.434
	Does not assume equal variances	1	-8.0000	1.97484	-4.051	11.545	.002
		2	-5.5000	1.25499	-4.383	7.022	.003
		3	-1.0000	1.37840	-.725	4.443	.505

$\hat{\psi}_{H1}: \quad t(11.54) = 4.05, p = .002$

$\hat{\psi}_{H2}: \quad t(7.02) = 4.38, p = .003$

$\hat{\psi}_{H3}: \quad t(4.44) = 0.73, p = .50$

  - Remember, this Welch correction only corrects for unequal variances and does not correct or adjust for non-normality.

- Try a non-parametric, rank-based alternative
  - Pair-wise tests can be conducted using the Mann-Whitney U test (or an ANOVA on the ranked data).
  - However, complex comparisons should be avoided! Because ranked data are ordinal data, we should not average (or take any linear combination) across groups.

  - A comparison of Mann-Whitney U pairwise contrasts with ANOVA by ranks approach

    $\psi_1 : \mu_1 = \mu_2$ $\qquad\qquad$ $\psi_2 : \mu_2 = \mu_3$

    $c_1 : (-1,1,0,0)$ $\qquad\qquad$ $c_2 : (0,-1,1,0)$

    - Mann-Whitney U pairwise contrasts:

```
NPAR TESTS
 /M-W= trials   BY reward(1 2).
```

Test Statistics

	TRIALS
Mann-Whitney U	9.500
Wilcoxon W	24.500
Z	-.638
Asymp. Sig. (2-tailed)	.523
Exact Sig. [2*(1-tailed Sig.)]	.548[a]

a. Not corrected for ties.

```
NPAR TESTS
 /M-W= trials   BY reward(2 3).
```

Test Statistics

	TRIALS
Mann-Whitney U	.000
Wilcoxon W	15.000
Z	-2.652
Asymp. Sig. (2-tailed)	.008
Exact Sig. [2*(1-tailed Sig.)]	.008[a]

a. Not corrected for ties.

- ANOVA by ranks approach:

```
RANK  VARIABLES=trials.
ONEWAY rtrials BY reward
  /CONT= -1 1 0 0
  /CONT= 0 1 -1 0.
```

Contrast Tests

		Contrast	Value of Contrast	Std. Error	t	df	Sig. (2-tailed)
RANK of TRIALS	Assume equal variances	1	-1.30000	2.391129	-.544	16	.594
		2	-8.80000	2.391129	-3.680	16	.002
	Does not assume equal variances	1	-1.30000	2.230471	-.583	5.397	.584
		2	-8.80000	2.173707	-4.048	4.954	.010

Contrast	Approach Mann-Whitney U	ANOVA by Ranks
$\mu_1 = \mu_2$	z = -0.638, p = .523	t(16) = -0.544, p = .594
$\mu_2 = \mu_3$	z = -2.652, p = .008	t(16) = -3.680, p = .002

- A rank modification of ANOVA is easy to use but:
  - Not much theoretical work has been done on this type of test.
  - This approach is probably not valid for multi-factor ANOVA.
  - This approach is likely to be trouble for complex comparisons.
  - Remember that the conclusions you draw are on the ranks, and not on the observed values!

7. Effect sizes for a single contrast
   - For pairwise contrasts, you can use Hedges's *g:*

$$g = \frac{\left|\bar{X}_1 - \bar{X}_2\right|}{\hat{\sigma}} = \frac{\left|\bar{X}_1 - \bar{X}_2\right|}{\sqrt{MSW}}$$

   - In the general case there are several options
     - Use omega squared (ω^2)

$$\hat{\omega}^2 = \frac{SS\hat{\psi} - MSW}{SST + MSW}$$

$\omega^2 = .01$	small effect size
$\omega^2 = .06$	medium effect size
$\omega^2 = .15$	large effect size

ω^2 has an easy interpretation: it is the percentage of the variance in the dependent variable (in the population) that is accounted for by the contrast

- Treat the complex comparison as a comparison between two groups and use Hedges's g, but we need the sum of the contrast coefficients to equal 2:

$$g = \frac{\hat{\psi}}{\sqrt{MSW}} \qquad \text{where} \qquad \sum |a_i| = 2$$

  - Any contrast can be considered to be a comparison between two groups. We can use the mean of those two groups to compute a d.

$$\psi_1 = \mu_1 - \frac{1}{4}\mu_2 - \frac{1}{4}\mu_3 - \frac{1}{4}\mu_4 - \frac{1}{4}\mu_5$$
$$\psi_2 = \frac{1}{2}\mu_2 + \frac{1}{2}\mu_3 - \frac{1}{2}\mu_4 - \frac{1}{2}\mu_5$$

  - ψ_1 is a comparison between group1 and the average of groups 2-5

$$H_0 : \mu_1 = \frac{\mu_2 + \mu_3 + \mu_4 + \mu_5}{4}$$

$$g = \frac{\overline{X}_1 - \frac{\overline{X}_2 + \overline{X}_3 + \overline{X}_4 + \overline{X}_5}{4}}{\sqrt{MSW}} = \frac{\hat{\psi}_1}{\sqrt{MSW}}$$

  - ψ_2 is a comparison between the average of groups 2 and 3 and the average of groups 4 and 5

$$H_0 : \frac{\mu_2 + \mu_3}{2} = \frac{\mu_4 + \mu_5}{2}$$

$$g = \frac{\frac{\overline{X}_2 + \overline{X}_3}{2} - \frac{\overline{X}_4 + \overline{X}_5}{2}}{\sqrt{MSW}} = \frac{\hat{\psi}_2}{\sqrt{MSW}}$$

  - Interpretation of this g is the same as the g for two groups, but you must be able to interpret the contrast as a comparison between two groups.
  - For example, polynomial contrasts cannot be considered comparisons between two groups. Thus, g is not appropriate for polynomial contrasts.

- Compute an r measure of effect size:

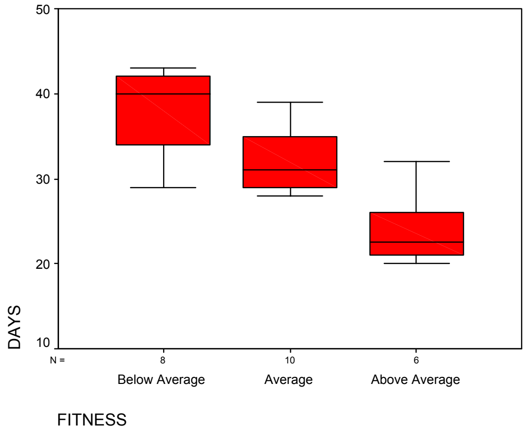
$$r = \sqrt{\frac{F_{contrast}}{F_{contrast} + df_{within}}} = \sqrt{\frac{t^2_{contrast}}{t^2_{contrast} + df_{within}}}$$

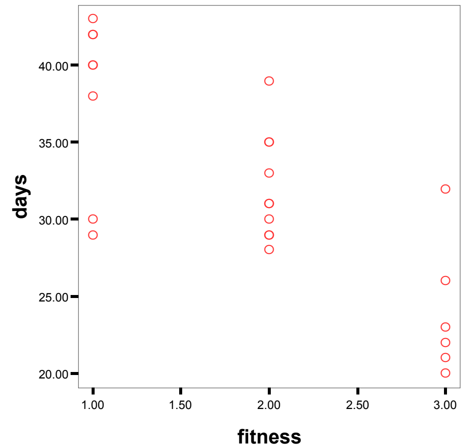
  - r is interpretable as the (partial) correlation between the group means and the contrast values, controlling for non-contrast variability.

8. An example
   - Rehabilitation Example. We have a sample of 24 male participants between the age of 18 and 30 who have all undergone corrective knee surgery in the past year. We would like to investigate the relationship between prior physical fitness status (below average, average, above average) and the number of days required for successful completion of physical therapy.

Prior physical fitness status		
Below Average	Average	Above Average
29	30	26
42	35	32
38	39	21
40	28	20
43	31	23
40	31	22
30	29	
42	35	
	29	
	33	

- We would like to test if:
  - Above average participants complete therapy faster than other groups
  - Average participants complete therapy faster than below average participants
  - Average participants complete therapy slower than above average participants





- We need to convert the hypotheses to contrast coefficients

  - Above average participants complete therapy faster than other groups

$$H_0 : \mu_3 = \frac{\mu_1 + \mu_2}{2}$$

$$\psi_1 = -\frac{1}{2}\mu_1 - \frac{1}{2}\mu_2 + \mu_3 \qquad c = \left(-\frac{1}{2}, -\frac{1}{2}, 1\right)$$

  - Average participants complete therapy faster than below average participants

$$H_0 : \mu_2 = \mu_1$$

$$\psi_2 = -\mu_1 + \mu_2 \qquad c = (-1, 1, 0)$$

  - Average participants complete therapy slower than above average participants

$$H_0 : \mu_2 = \mu_3$$

$$\psi_3 = -\mu_2 + \mu_3 \qquad c = (0, -1, 1)$$

- Are these three contrasts an orthogonal set?

  - With 3 groups, we can only have 2 orthogonal contrasts
    If we had equal sample sizes, then $\psi_1 \perp \psi_2$
    With unequal n we do not have an orthogonal set of contrasts

- Conduct significance tests for these contrasts

```
ONEWAY days BY fitness
 /STAT desc
 /CONTRAST = -.5,-.5,1
 /CONTRAST = -1, 1, 0
 /CONTRAST = 0, -1, 1.
```

Descriptives

DAYS

	N	Mean	Std. Deviation	Std. Error	95% Confidence Interval for Mean Lower Bound	95% Confidence Interval for Mean Upper Bound
Below Average	8	38.0000	5.47723	1.93649	33.4209	42.5791
Average	10	32.0000	3.46410	1.09545	29.5219	34.4781
Above Average	6	24.0000	4.42719	1.80739	19.3540	28.6460
Total	24	32.0000	6.87782	1.40393	29.0958	34.9042

ANOVA

DAYS

	Sum of Squares	df	Mean Square	F	Sig.
Between Groups	672.000	2	336.000	16.962	.000
Within Groups	416.000	21	19.810		
Total	1088.000	23			

Contrast Coefficients

Contrast	FITNESS Below Average	FITNESS Average	FITNESS Above Average
1	-.5	-.5	1
2	-1	1	0
3	0	-1	1

Contrast Tests

		Contrast	Value of Contrast	Std. Error	t	df	Sig. (2-tailed)
DAYS	Assume equal variances	1	-11.0000	2.10140	-5.235	21	.000
		2	-6.0000	2.11119	-2.842	21	.010
		3	-8.0000	2.29838	-3.481	21	.002
	Does not assume equal variances	1	-11.0000	2.12230	-5.183	8.938	.001
		2	-6.0000	2.22486	-2.697	11.297	.020
		3	-8.0000	2.11345	-3.785	8.696	.005

- Compute a measure of effect size for each contrast

$$\hat{\omega}^2 = \frac{SS\hat{\psi} - MSW}{SST + MSW}$$

  - We need to compute the SS for each contrast

$$SS\hat{\psi} = \frac{\hat{\psi}^2}{\sum \frac{c_i^2}{n_i}}$$

$$SS\hat{\psi}_1 = \frac{(-11)^2}{\frac{\left(-\frac{1}{2}\right)^2}{8} + \frac{\left(-\frac{1}{2}\right)^2}{10} + \frac{(1)^2}{6}} = \frac{121}{.2229} = 542.80$$

$$SS\hat{\psi}_2 = \frac{(-6)^2}{\frac{(-1)^2}{8} + \frac{(1)^2}{10} + \frac{(0)^2}{6}} = \frac{36}{.225} = 160$$

$$SS\hat{\psi}_3 = \frac{(-8)^2}{0 + \frac{(-1)^2}{10} + \frac{(1)^2}{6}} = \frac{64}{.2667} = 240$$

  - Now compute omega squared

$$\hat{\psi}_1 : \hat{\omega}^2 = \frac{542.80 - 19.81}{1088 + 19.81} = .472$$

$$\hat{\psi}_2 : \hat{\omega}^2 = \frac{160 - 19.81}{1088 + 19.81} = .127$$

$$\hat{\psi}_3 : \hat{\omega}^2 = \frac{240 - 19.81}{1088 + 19.81} = .199$$

- OR Compute an *r* measure of effect size for each contrast:

$$r = \sqrt{\frac{F_{contrast}}{F_{contrast} + df_{within}}} = \sqrt{\frac{t^2_{contrast}}{t^2_{contrast} + df_{within}}}$$

$$\hat{\psi}_1 : r_1 = \sqrt{\frac{5.235^2}{5.235^2 + 21}} = .75$$

$$\hat{\psi}_2 : r_2 = \sqrt{\frac{2.842^2}{2.842^2 + 21}} = .53$$

$$\hat{\psi}_3 : r_2 = \sqrt{\frac{3.481^2}{3.481^2 + 21}} = .61$$

- Report the results

  $\hat{\psi}_1 : t(21) = -5.24, p < .01, \omega^2 = .47$

  $\hat{\psi}_2 : t(21) = -2.84, p = .01, \omega^2 = .13$

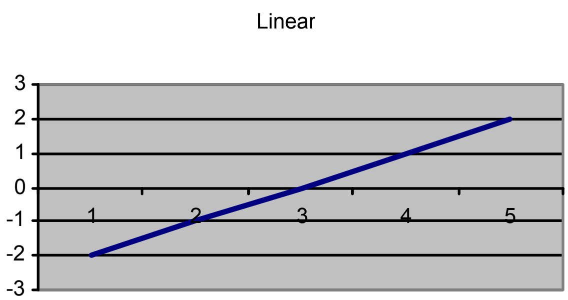
  $\hat{\psi}_3 : t(21) = -3.48, p < .01, \omega^2 = .20$

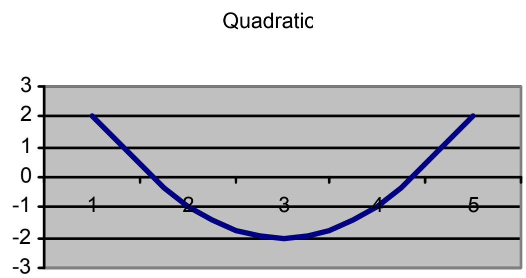
  - In your results section, you need to say in English (not in statistics or symbols) what each contrast is testing
  - In general, it is not necessary to report the value of the contrast or the contrast coefficients used

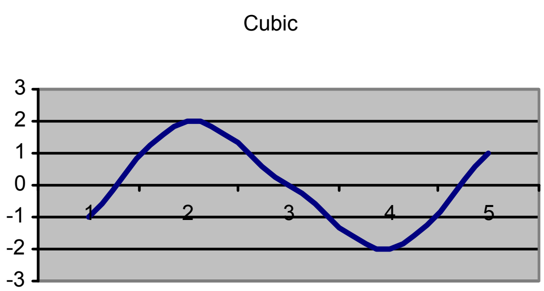
. . . A contrast revealed that above average individuals recovered faster than all other individuals, $t(21) = -5.24, p < .01, \omega^2 = .47$. Pairwise tests also revealed that average individuals completed therapy faster than below average individuals, $t(21) = -2.84, p = .01, \omega^2 = .13$, and that above average individuals completed therapy faster than average participants, $t(21) = -3.48, p < .01, \omega^2 = .20$.

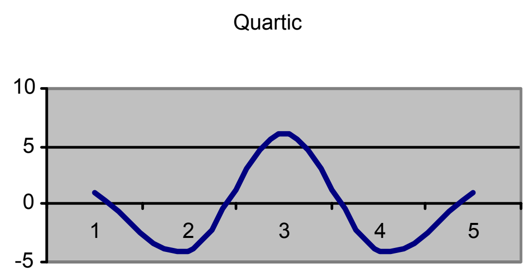
9. Polynomial Trend Contrasts
   - Trend contrasts are a specific kind of orthogonal contrasts that may be of interest for certain designs.
   - Tests for trends are used only for comparing quantitative (ordered) independent variables.
     - IV = 10mg, 20mg, 30mg, 40mg of a drug
   - Trend contrasts are used to explore polynomial trends in the data

Trend	Order of Polynomial	# of Bends	Shape
Linear	1st	0	Straight Line
Quadratic	2nd	1	U-shaped
Cubic	3rd	2	Wave
Quartic	4th	3	Wave
Etc.			



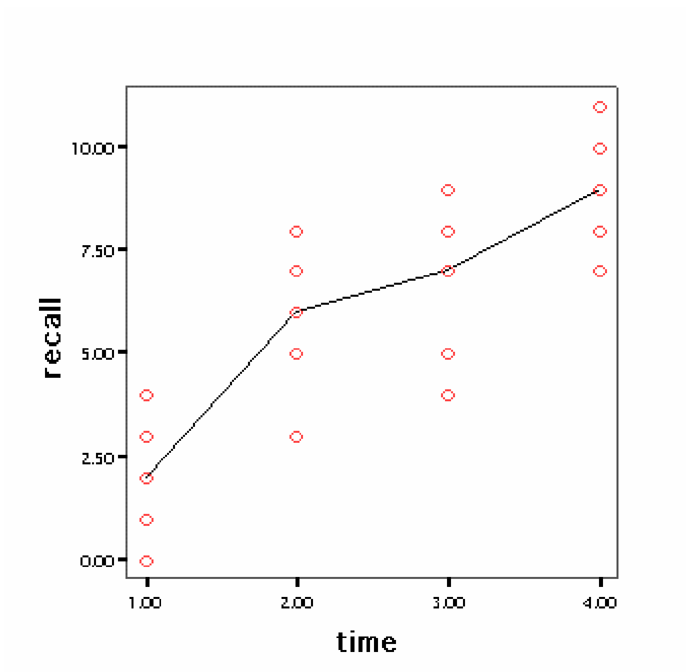






- A Memory Example #1

Study Time			
1 Minute	2 Minutes	3 Minutes	4 Minutes
2	6	5	11
3	8	7	10
1	5	9	7
2	3	4	9
0	7	9	8
4	7	8	9
2	**6**	**7**	**9**



- It looks like there might be a linear trend in the data
- To test for trends, tables of orthogonal trend contrasts have been computed. For a=4, we can have 3 orthogonal contrasts

	c_1	c_2	c_3	c_4
Linear	-3	-1	1	3
Quadratic	1	-1	-1	1
Cubic	-1	3	-3	1

- To use these values, the levels of the IV need to be <u>equally spaced</u> and the <u>cell sizes must be equal</u>

- To compute the trend contrasts, we use the orthogonal trend contrasts and the usual procedure for computing and testing contrasts:

    - For the linear contrast, use c = (-3,-1, 1, 3)

$$\psi_{linear} = -3\mu_1 - \mu_2 + \mu_3 + 3\mu_4$$
$$\hat{\psi}_{linear} = -3\bar{X}_1 - 1\bar{X}_2 + \bar{X}_3 + 3\bar{X}_4 = -3(2) - (6) + (8) + 3(9) = 23$$
$$SS(\hat{\psi}_{linear}) = \frac{(23)^2}{\frac{(-3)^2}{6} + \frac{(-1)^2}{6} + \frac{(1)^2}{6} + \frac{(3)^2}{6}} = 158.7$$

    - For the quadratic contrast, use c = (1, -1, -1, 1)

$$\psi_{quadratic} = \mu_1 - \mu_2 - \mu_3 + \mu_4$$
$$\hat{\psi}_{quadratic} = \bar{X}_1 - \bar{X}_2 - \bar{X}_3 + \bar{X}_4 = 2 - 6 - 8 + 9 = -3$$
$$SS(\hat{\psi}_{quadratic}) = \frac{(-3)^2}{\frac{(1)^2}{6} + \frac{(-1)^2}{6} + \frac{(-1)^2}{6} + \frac{(1)^2}{6}} = 13.50$$

    - For the cubic contrast, use c = (-1,3, -3, 1)

$$\psi_{cubic} = -\mu_1 + 3\mu_2 - 3\mu_3 + \mu_4$$
$$\hat{\psi}_{cubic} = -\bar{X}_1 + 3\bar{X}_2 - 3\bar{X}_3 + \bar{X}_4 = -2 + 3(6) - 3(8) + 9 = 1$$
$$SS(\hat{\psi}_{cubic}) = \frac{(1)^2}{\frac{(-1)^2}{6} + \frac{(3)^2}{6} + \frac{(-3)^2}{6} + \frac{(1)^2}{6}} = 0.30$$

- Comments about trend contrasts
    - These contrasts are orthogonal (when *n*s are equal), so it is possible to have any combination of effects (or lack of effects)
    - Because the sets of weights are not equally scaled, you cannot compare the strength of effects simply by inspecting the value of the contrast.

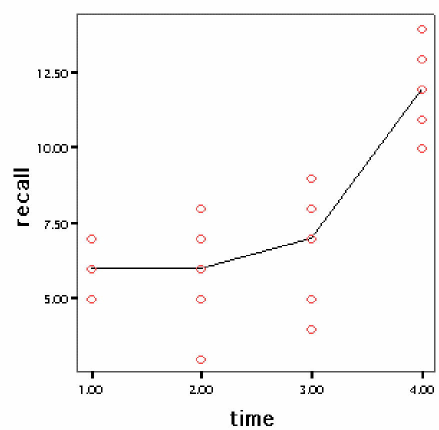
    - Some people place an additional constraint on the contrast weights:

$$\sum_{i=1}^{a} |c_i| = 2$$

        - When the sum of the absolute value of the contrast values is not constant across contrasts (as with the trend contrasts), then you CAN NOT compare contrast values. You can only compare sums of squares and measures of effect size.

- A Memory Example #2

Study Time			
1 Minute	2 Minutes	3 Minutes	4 Minutes
6	6	5	14
7	8	7	13
5	5	9	10
6	3	4	12
4	7	9	11
8	7	8	12
6	**6**	**7**	**12**



- Looks like we have a linear effect with some quadratic

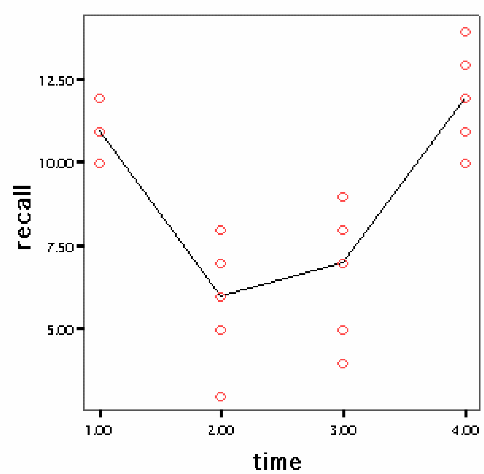
$$\psi_{linear} = -3\mu_1 - \mu_2 + \mu_3 + 3\mu_4$$
$$\hat{\psi}_{linear} = -3\bar{X}_1 - 1\bar{X}_2 + \bar{X}_3 + 3\bar{X}_4 = -3(6) - (6) + (7) + 3(12) = 19$$
$$SS(\hat{\psi}_{linear}) = \frac{(19)^2}{\frac{(-3)^2}{6} + \frac{(-1)^2}{6} + \frac{(1)^2}{6} + \frac{(3)^2}{6}} = 108.3$$

$$\psi_{quadratic} = \mu_1 - \mu_2 - \mu_3 + \mu_4$$
$$\hat{\psi}_{quadratic} = \bar{X}_1 - \bar{X}_2 - \bar{X}_3 + \bar{X}_4 = 6 - 6 - 7 + 12 = 5$$
$$SS(\hat{\psi}_{quadratic}) = \frac{(5)^2}{\frac{(1)^2}{6} + \frac{(-1)^2}{6} + \frac{(-1)^2}{6} + \frac{(1)^2}{6}} = 37.50$$

$$\psi_{cubic} = -\mu_1 + 3\mu_2 - 3\mu_3 + \mu_4$$
$$\hat{\psi}_{cubic} = -\bar{X}_1 + 3\bar{X}_2 - 3\bar{X}_3 + \bar{X}_4 = -6 + 3(6) - 3(7) + 12 = 3$$
$$SS(\hat{\psi}_{cubic}) = \frac{(3)^2}{\frac{(-1)^2}{6} + \frac{(3)^2}{6} + \frac{(-3)^2}{6} + \frac{(1)^2}{6}} = 2.70$$

- A Memory Example #3

Study Time			
1 Minute	2 Minutes	3 Minutes	4 Minutes
11	6	5	14
12	8	7	13
10	5	9	10
11	3	4	12
10	7	9	11
12	7	8	12
11	**6**	**7**	**12**



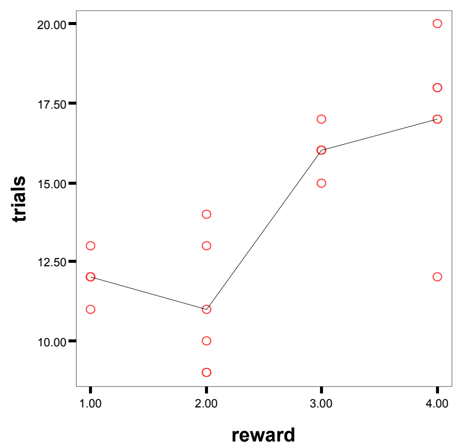
- Looks like we have a quadratic effect

$$\psi_{linear} = -3\mu_1 - \mu_2 + \mu_3 + 3\mu_4$$
$$\hat{\psi}_{linear} = -3\bar{X}_1 - 1\bar{X}_2 + \bar{X}_3 + 3\bar{X}_4 = -3(11) - (6) + (7) + 3(12) = 4$$
$$SS(\hat{\psi}_{linear}) = \frac{(4)^2}{\frac{(-3)^2}{6} + \frac{(-1)^2}{6} + \frac{(1)^2}{6} + \frac{(3)^2}{6}} = 4.8$$

$$\psi_{quadratic} = \mu_1 - \mu_2 - \mu_3 + \mu_4$$
$$\hat{\psi}_{quadratic} = \bar{X}_1 - \bar{X}_2 - \bar{X}_3 + \bar{X}_4 = 6 - 6 - 7 + 12 = 10$$
$$SS(\hat{\psi}_{quadratic}) = \frac{(10)^2}{\frac{(1)^2}{6} + \frac{(-1)^2}{6} + \frac{(-1)^2}{6} + \frac{(1)^2}{6}} = 150$$

$$\psi_{cubic} = -\mu_1 + 3\mu_2 - 3\mu_3 + \mu_4$$
$$\hat{\psi}_{cubic} = -\bar{X}_1 + 3\bar{X}_2 - 3\bar{X}_3 + \bar{X}_4 = -11 + 3(6) - 3(7) + 12 = -2$$
$$SS(\hat{\psi}_{cubic}) = \frac{(-2)^2}{\frac{(-1)^2}{6} + \frac{(3)^2}{6} + \frac{(-3)^2}{6} + \frac{(1)^2}{6}} = 1.20$$

- Statistical tests for trend analysis: Reanalyzing the learning example
  - Our learning example is perfectly suited for a trend analysis (Why?)
  - When we initially analyzed these data, we selected one set of orthogonal contrasts, but there are many possible sets of orthogonal contrasts, including the trend contrasts



  - For $a = 4$, we can test for a linear, a quadratic, and a cubic trend

	c_1	c_2	c_3	c_4
Linear	-3	-1	1	3
Quadratic	1	-1	-1	1
Cubic	-1	3	-3	1

$$\begin{aligned}\hat{\psi}_{linear} &= -3\overline{X}_1 - \overline{X}_2 + \overline{X}_3 + 3\overline{X}_4 \\ &= -3(12) - (11) + (16) + 3(17) \\ &= 20\end{aligned}$$

$$SS(\hat{\psi}_{lin}) = \frac{(20)^2}{\frac{(-3)^2}{5} + \frac{(-1)^2}{5} + \frac{(1)^2}{5} + \frac{(3)^2}{5}} = 100$$

$$\begin{aligned}\hat{\psi}_{quadratic} &= \overline{X}_1 - \overline{X}_2 - \overline{X}_3 + \overline{X}_4 \\ &= (12) - (11) - (16) + (17) \\ &= 2\end{aligned}$$

$$SS(\hat{\psi}_{quad}) = \frac{(2)^2}{\frac{(1)^2}{5} + \frac{(-1)^2}{5} + \frac{(-1)^2}{5} + \frac{(1)^2}{5}} = 5$$

$$\begin{aligned}\hat{\psi}_{cubic} &= -\overline{X}_1 + 3\overline{X}_2 - 3\overline{X}_3 + \overline{X}_4 \\ &= -(12) + 3(11) - 3(16) + (17) \\ &= -10\end{aligned}$$

$$SS(\hat{\psi}_{cubic}) = \frac{(-10)^2}{\frac{(-1)^2}{5} + \frac{(3)^2}{5} + \frac{(-3)^2}{5} + \frac{(1)^2}{5}} = 25$$

- Rather than determine significance by hand, we can use ONEWAY:

```
ONEWAY  trials BY reward
  /CONT -3, -1, 1, 3
  /CONT 1, -1, -1, 1
  /CONT -1, 3, -3, 1.
```

Contrast Coefficients

	REWARD			
Contrast	1.00	2.00	3.00	4.00
1	-3	-1	1	3
2	1	-1	-1	1
3	-1	3	-3	1

Contrast Tests

		Contrast	Value of Contrast	Std. Error	t	df	Sig. (2-tailed)
TRIALS	Assume equal variances	1	20.0000	3.93700	5.080	16	.000
		2	2.0000	1.76068	1.136	16	.273
		3	-10.0000	3.93700	-2.540	16	.022

- We find evidence for significant linear and cubic trends

$\hat{\psi}_{linear} : t(16) = 5.08, p < .01, r = .79$

$\hat{\psi}_{quadratic} : t(16) = 1.14, p = .27, r = .27$

$\hat{\psi}_{cubic} : t(16) = -2.54, p = .02, r = .54$

- To complete the ANOVA table, we need the sums of squares for each contrast

$$SS\hat{\psi} = \frac{\hat{\psi}^2}{\sum \frac{c_i^2}{n_i}}$$

$$SS\hat{\psi}_{linear} = 100$$
$$SS\hat{\psi}_{quad} = 5$$
$$SS\hat{\psi}_{cubic} = 25$$

$$SS\hat{\psi}_{linear} + SS\hat{\psi}_{quadratic} + SS\hat{\psi}_{cubic} = 100 + 5 + 25 = 130 = SSB$$

ANOVA

Source of Variation	*SS*	*df*	*MS*	*F*	*P-value*
Between Groups	130	3	43.33333	11.1828	0.000333
$\hat{\psi}_{lin}$	100	1	100	25.806	0.000111
$\hat{\psi}_{quad}$	5	1	5	1.290	0.272775
$\hat{\psi}_{cubic}$	25	1	25	6.452	0.021837
Within Groups	62	16	3.875		
Total	192	19			

- You can also directly ask for polynomial contrasts in SPSS
  - Method 1: ONEWAY

```
ONEWAY trials BY reward
  /POLYNOMIAL= 3.
```

    - After polynomial, enter the highest degree polynomial you wish to test.

ANOVA

TRIALS

			Sum of Squares	df	Mean Square	F	Sig.
Between Groups	(Combined)		130.000	3	43.333	11.183	.000
	Linear Term	Contrast	100.000	1	100.000	25.806	.000
		Deviation	30.000	2	15.000	3.871	.043
	Quadratic Term	Contrast	5.000	1	5.000	1.290	.273
		Deviation	25.000	1	25.000	6.452	.022
	Cubic Term	Contrast	25.000	1	25.000	6.452	.022
Within Groups			62.000	16	3.875		
Total			192.000	19			

  - Advantages of the ONEWAY method for polynomial contrasts:
    - It utilizes the easiest oneway ANOVA command
    - It gives you the sums of squares of the contrast
    - It uses the spacing of the IV in the data (Be careful!)
    - It gives you the "deviation" test (to be explained later)

  - Disadvantages of the ONEWAY method for polynomial contrasts:
    - You can not see the value of the contrast or the contrast coefficients

o <u>Method 2</u>: UNIANOVA

```
UNIANOVA  trials  BY reward
  /CONTRAST (reward)=Polynomial
  /PRINT = test(lmatrix).
```

Contrast Results (K Matrix)

REWARD Polynomial Contrast[a]			Dependent Variable TRIALS
Linear	Contrast Estimate		4.472
	Hypothesized Value		0
	Difference (Estimate - Hypothesized)		4.472
	Std. Error		.880
	Sig.		.000
	95% Confidence Interval for Difference	Lower Bound	2.606
		Upper Bound	6.338
Quadratic	Contrast Estimate		1.000
	Hypothesized Value		0
	Difference (Estimate - Hypothesized)		1.000
	Std. Error		.880
	Sig.		.273
	95% Confidence Interval for Difference	Lower Bound	-.866
		Upper Bound	2.866
Cubic	Contrast Estimate		-2.236
	Hypothesized Value		0
	Difference (Estimate - Hypothesized)		-2.236
	Std. Error		.880
	Sig.		.022
	95% Confidence Interval for Difference	Lower Bound	-4.102
		Upper Bound	-.370

a. Metric = 1.000, 2.000, 3.000, 4.000

Test Results

Dependent Variable: TRIALS

Source	Sum of Squares	df	Mean Square	F	Sig.
Contrast	130.000	3	43.333	11.183	.000
Error	62.000	16	3.875		

Contrast Coefficients (L' Matrix)

Parameter	REWARD Polynomial Contrast[a] Linear	Quadratic	Cubic
Intercept	.000	.000	.000
[REWARD=1.00]	-.671	.500	-.224
[REWARD=2.00]	-.224	-.500	.671
[REWARD=3.00]	.224	-.500	-.671
[REWARD=4.00]	.671	.500	.224

The default display of this matrix is the transpose of the corresponding L matrix.

a. Metric = 1.000, 2.000, 3.000, 4.000

- This is the matrix of trend coefficients used by SPSS to calculate the contrasts.

SPSS coefficients

$c_1 = (-.671, -.224, .224, .671)$

$c_2 = (.5, -.5, -.5, .5)$

$c_3 = (-.224, .671, -.671 .224)$

SPSS coefficients X 6

$c_1 = (-4, -1, 1, 4)$

$c_2 = (3, -3, -3, 3)$

$c_3 = (-1, 4, -4, 1)$

- You can check that these coefficients are orthogonal

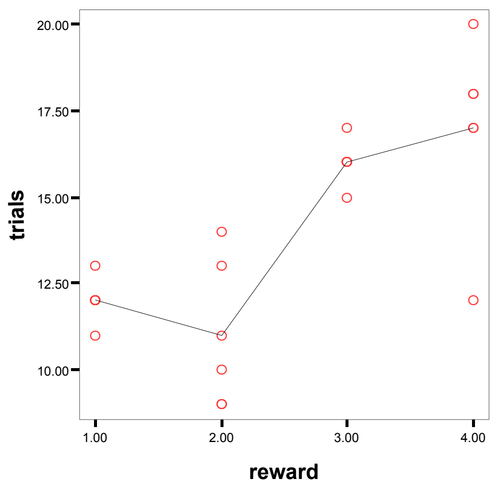
- Suppose the reward intervals were not equally spaced:

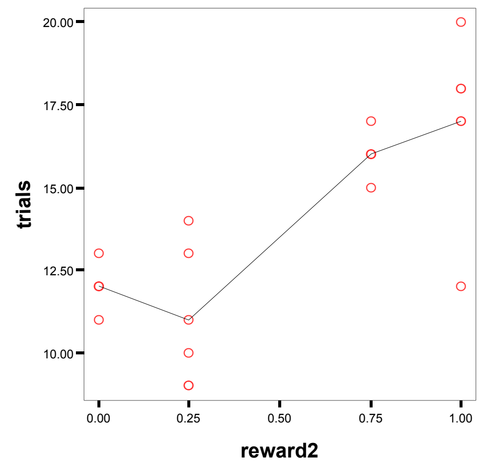
Level of Reward			
Constant (100%)	Frequent (75%)	Infrequent (25%)	Never (0%)

- Now we cannot use the tabled contrast values, because they require equal spacing between intervals

Equal spacing

Unequal spacing






```
UNIANOVA  trials  BY reward
  /CONTRAST (reward)=Polynomial (1, .75, .25, 0)
  /PRINT = test(lmatrix).
```

Contrast Results (K Matrix)

REWARD Polynomial Contrast[a]			Dependent Variable TRIALS
Linear	Contrast Estimate		-4.743
	Hypothesized Value		0
	Difference (Estimate - Hypothesized)		-4.743
	Std. Error		.880
	Sig.		.000
	95% Confidence Interval for Difference	Lower Bound	-6.610
		Upper Bound	-2.877
Quadratic	Contrast Estimate		1.000
	Hypothesized Value		0
	Difference (Estimate - Hypothesized)		1.000
	Std. Error		.880
	Sig.		.273
	95% Confidence Interval for Difference	Lower Bound	-.866
		Upper Bound	2.866
Cubic	Contrast Estimate		1.581
	Hypothesized Value		0
	Difference (Estimate - Hypothesized)		1.581
	Std. Error		.880
	Sig.		.091
	95% Confidence Interval for Difference	Lower Bound	-.285
		Upper Bound	3.447

a. Metric = 1.000, .750, .250, .000

* Now only the linear trend is significant

* SPSS calculates a set of orthogonal trend contrasts, based on the spacing you provide. Here they are:

Contrast Coefficients (L' Matrix)

Parameter	REWARD Polynomial Contrast[a] Linear	Quadratic	Cubic
Intercept	.000	.000	.000
[REWARD=1.00]	.632	.500	.316
[REWARD=2.00]	.316	-.500	-.632
[REWARD=3.00]	-.316	-.500	.632
[REWARD=4.00]	-.632	.500	-.316

The default display of this matrix is the transpose of the corresponding L matrix.

a. Metric = 1.000, .750, .250, .000

- Advantages of the UNIANOVA method:
  * It is the only way SPSS gives you confidence intervals for a contrast
    (But remember, the width of CIs depends on the contrast values)
  * It allows you to deal with unequally spaced intervals
    (It assumes equal spacing unless you tell it otherwise; no matter how you have the data coded!)
  * It will print the contrast values SPSS uses

- Disadvantages of the UNIANOVA method:
  * It does not print out a test statistic or the degrees of freedom of the test!?!

* Remember, for a one-way design, you can obtain a test of any contrast by using the ONEWAY command and entering the values for each contrast. With ONEWAY method, you know exactly how the contrast is being computed and analyzed

10. Simultaneous significance tests on multiple orthogonal contrasts

- Sometimes you may wish to test the significance of several contrasts in a single omnibus test

- Example #1
  - We would like to compare the effect of four drugs on body temperature. To test these drugs, we randomly assign people to receive one of the four drugs. After a period of time, we record each participant's body temperature.

Drug A	Drug B	Drug C	Drug D
95.4	95.5	94.7	96.1
94.8	96.5	95.0	95.5
95.0	96.5	94.9	96.4
95.2	96.1	94.6	94.8
95.6	95.9	95.3	95.7

  - Our hypothesis was that Drug B would result in a higher body temperature than the other drugs

$$\psi_1 = -1\mu_A + 3\mu_B - 1\mu_C - 1\mu_D$$

  - We also wanted to know if the other drugs (A, C, D) differed in their effect on body temperature

$$H_0 : \mu_A = \mu_C = \mu_D \qquad H_1 : \text{The three means are not equal}$$

    - Note that this hypothesis is an omnibus hypothesis!

Report

TEMP

DRUG	Mean	N	Std. Deviation
A	95.2000	5	.31623
B	96.1000	5	.42426
C	94.9000	5	.27386
D	95.7000	5	.61237
Total	95.4750	20	.61377

- Step 1: Obtain *SSB*, *SST*, and *MSW*

ANOVA

TEMP

	Sum of Squares	df	Mean Square	F	Sig.
Between Groups	4.237	3	1.412	7.740	.002
Within Groups	2.920	16	.183		
Total	7.157	19			

- Step 2: Test $\psi_1 = -1\mu_A + 3\mu_B - 1\mu_C - 1\mu_D$

$$\hat{\psi}_1 = -1(95.2) + 3(96.1) - 1(94.9) - 1(97.5) = 2.5$$

$$SS\hat{\psi}_1 = \frac{(2.5)^2}{\frac{(-1)^2}{5} + \frac{(3)^2}{5} + \frac{(-1)^2}{5} + \frac{(-1)^2}{5}} = \frac{6.25}{2.4} = 2.6042$$

ANOVA

Source of Variation	SS	df	MS	F	P-value
Between Groups	4.237	3	1.412	7.740	.002
ψ_1	2.6042	1	2.6042	14.2694	0.0017
Within Groups	2.92	16	.1825		
Total	7.1575	19			

- Step 3: Test $H_0 : \mu_A = \mu_C = \mu_D$
  - The trick is to remember that an omnibus ANOVA test *m* means is equal to the simultaneous test on <u>any set of (*m-1*) orthogonal contrasts</u>
  - We can then combine these orthogonal contrasts in a single omnibus F-test:

$$F_{comb}(m-1, dfw) = \frac{\frac{SSC_1 + ... + SSC_{m-1}}{m-1}}{MSW}$$

- We need to choose any 2 contrasts so long as we have an orthogonal set of three contrasts (including the contrast associated with the first hypothesis):

  $c_1 : (-1,3,-1,-1)$

  $c_2 : (-1,0,-1,2)$

  $c_3 : (-1,0,1,0)$

- This set will work because all three contrasts are orthogonal. Now, let's compute the simultaneous F-test of these two contrasts.

$$\hat{\psi}_2 = -1(95.2) + 0 - 1(94.9) + 2(97.5) = 1.3$$

$$SS\hat{\psi}_2 = \frac{(1.3)^2}{\frac{(-1)^2}{5} + \frac{(0)^2}{5} + \frac{(-1)^2}{5} + \frac{(2)^2}{5}} = \frac{1.69}{1.2} = 1.408$$

$$\hat{\psi}_3 = -1(95.2) + 0 + 1(94.9) + 0(97.5) = -0.3$$

$$SS\hat{\psi}_3 = \frac{(-0.3)^2}{\frac{(-1)^2}{5} + \frac{(0)^2}{5} + \frac{(1)^2}{5} + \frac{(0)^2}{5}} = \frac{.09}{.4} = .225$$

$$F_{comb}(m-1, dfw) = \frac{\frac{SSC_1 + ... + SSC_{m-1}}{m-1}}{MSW}$$

$$F_{comb}(2,16) = \frac{\frac{1.408 + 0.225}{2}}{.1825} = 4.47, p = .03$$

We reject H_0 and conclude that $\mu_A, \mu_C, and\ \mu_D$ are not all equal

ANOVA

Source of Variation	*SS*	*df*	*MS*	*F*	*P-value*
Between Groups	4.2375	3	1.412	7.740	.002
ψ_1	2.6042	1	2.6042	14.2694	0.0017
$\mu_A = \mu_C = \mu_D$ (ψ_2, ψ_3)	1.6333	2	0.8167	4.4748	.0286
Within Groups	2.92	16	.1825		
Total	7.1575	19			

- Note: We also could have computed the test of $\mu_A = \mu_C = \mu_D$ without directly computing the two orthogonal contrasts:
  - If we knew the combined sums of squares of these two contrasts then we could fill in the remainder of the ANOVA table.
  - But we do know the combined sums of squares for the remaining two contrasts (so long as all the contrasts are orthogonal)!

ANOVA					
Source of Variation	*SS*	*df*	*MS*	*F*	*P-value*
Between Groups	4.2375	3	1.412	7.740	.002
ψ_1	2.6042	1	2.6042	14.2694	0.0017
$\mu_A = \mu_C = \mu_D$		2			
Within Groups	2.92	16	.1825		
Total	7.1575	19			

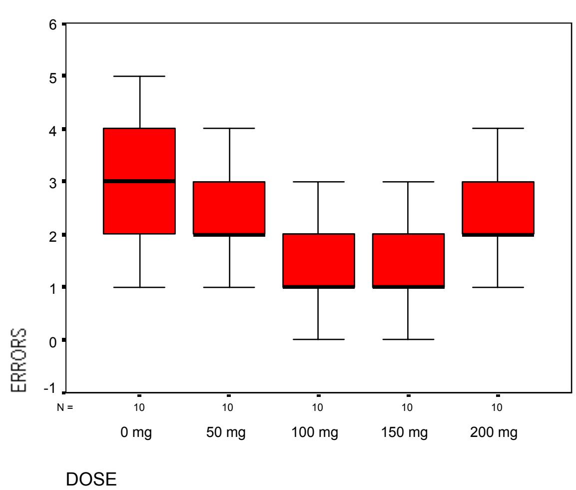
$$SSB = SS\psi_1 + SS\psi_2 + SS\psi_3$$
$$SS\psi_2 + SS\psi_3 = SSB - SS\psi_1 = 4.2375 - 2.6042 = 1.6333$$

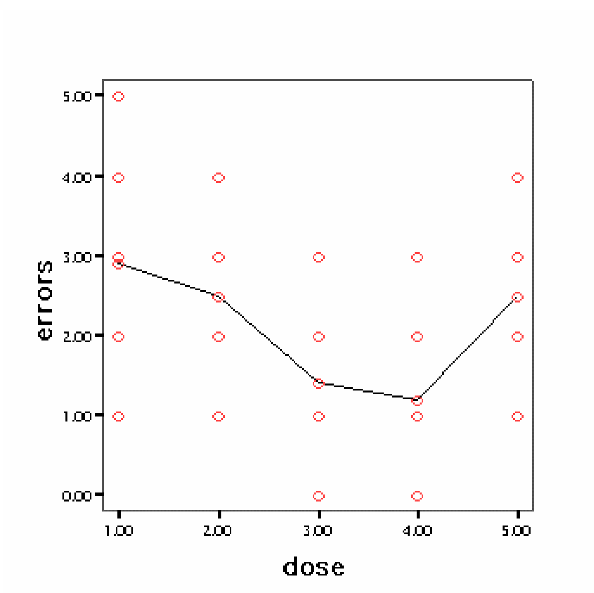
- We can substitute $SS\psi_2 + SS\psi_3$ into the table and compute the F-test as we did previous (except in this case, we never identified or computed the two additional contrasts to complete the orthogonal set).

ANOVA					
Source of Variation	*SS*	*df*	*MS*	*F*	*P-value*
Between Groups	4.2375	3	1.412	7.740	.002
ψ_1	2.6042	1	2.6042	14.2694	0.0017
$\mu_A = \mu_C = \mu_D$	1.6333	2	0.8167	4.4748	.0286
Within Groups	2.92	16	.1825		
Total	7.1575	19			

- Example #2
  - We want to examine the effect of caffeine on cognitive performance and attention. Participants are randomly assigned to one of 5 dosages of caffeine. In a subsequent proofreading task, we count the number of errors.

Dose of Caffeine				
0mg	50mg	100mg	150mg	200mg
2	2	0	1	2
4	3	1	0	3
5	4	3	2	4
3	2	1	1	4
2	2	1	1	2
1	1	2	2	1
3	2	2	1	2
3	2	1	0	3
2	3	1	1	2
4	4	2	3	2





Descriptives

ERRORS

	N	Mean	Std. Deviation	Std. Error	95% Confidence Interval for Mean	
					Lower Bound	Upper Bound
0 mg	10	2.9000	1.19722	.37859	2.0436	3.7564
50 mg	10	2.5000	.97183	.30732	1.8048	3.1952
100 mg	10	1.4000	.84327	.26667	.7968	2.0032
150 mg	10	1.2000	.91894	.29059	.5426	1.8574
200 mg	10	2.5000	.97183	.30732	1.8048	3.1952
Total	50	2.1000	1.16496	.16475	1.7689	2.4311

- We would like to test if there is a linear or a quadratic trend. We are not really interested in any higher order trends

- With equally spaced intervals, we can use the coefficients from the orthogonal polynomial table. With five groups, we can test up to four orthogonal polynomials

	c_1	c_2	c_3	c_4	c_5
Linear	-2	-1	0	1	2
Quadratic	2	-1	-2	-1	2
Cubic	-1	2	0	-2	1
Quartic	1	-4	6	-4	1

- Method 1: Let SPSS do all the work

```
ONEWAY errors BY dose
 /POLYNOMIAL= 2.
```

ANOVA

ERRORS

			Sum of Squares	df	Mean Square	F	Sig.
Between Groups	(Combined)		22.600	4	5.650	5.792	.001
	Linear Term	Contrast	4.410	1	4.410	4.521	.039
		Deviation	18.190	3	6.063	6.215	.001
	Quadratic Term	Contrast	13.207	1	13.207	13.538	.001
		Deviation	4.983	2	2.491	2.554	.089
Within Groups			43.900	45	.976		
Total			66.500	49			

- Under "Linear Term"
  CONTRAST is the test for the linear contrast: $F(1,45) = 4.52, p = .039$
  DEVIATION is the combined test for the quadratic, cubic, and quartic contrasts: $F(3,45) = 6.62, p = .001$

- Under "Quadratic Term"
  CONTRAST is the test for the quadratic contrast:
  $F(1,45) = 13.54, p = .001$
  DEVIATION is the combined test for the cubic and quartic contrasts
  $F(2,45) = 2.55, p = .089$

- Is it safe to report that there are no higher order trends?

- Method 2a: Let SPSS do some of the work

```
ONEWAY errors BY dose
 /CONT= -2 -1  0  1  2
 /CONT=  2 -1 -2 -1  2
 /CONT= -1  2  0 -2  1
 /CONT=  1 -4  6 -4  1.
```

Contrast Tests

		Contrast	Value of Contrast	Std. Error	t	df	Sig. (2-tailed)
ERRORS	Assume equal variances	1	-2.1000	.98770	-2.126	45	.039
		2	4.3000	1.16866	3.679	45	.001
		3	2.2000	.98770	2.227	45	.031
		4	-1.0000	2.61321	-.383	45	.704
	Does not assume equal variances	1	-2.1000	1.06301	-1.976	23.575	.060
		2	4.3000	1.18930	3.616	31.679	.001
		3	2.2000	.97639	2.253	28.573	.032
		4	-1.0000	2.37908	-.420	26.966	.678

- Method 2b: Let SPSS do some of the work

```
UNIANOVA errors BY dose
 /CONTRAST (dose)=POLYNOMIAL
 /PRINT=TEST(LMATRIX).
```

Contrast Coefficients (L' Matrix)

Parameter	DOSE Polynomial Contrast[a] Linear	Quadratic	Cubic	Order 4
Intercept	.000	.000	.000	.000
[DOSE=1.00]	-.632	.535	-.316	.120
[DOSE=2.00]	-.316	-.267	.632	-.478
[DOSE=3.00]	.000	-.535	.000	.717
[DOSE=4.00]	.316	-.267	-.632	-.478
[DOSE=5.00]	.632	.535	.316	.120

The default display of this matrix is the transpose of the corresponding L matrix.

a. Metric = 1.000, 2.000, 3.000, 4.000, 5.000

Contrast Results (K Matrix)

DOSE Polynomial Contrast[a]			Dependent Variable ERRORS
Linear	Contrast Estimate		-.664
	Hypothesized Value		0
	Difference (Estimate - Hypothesized)		-.664
	Std. Error		.312
	Sig.		.039
	95% Confidence Interval for Difference	Lower Bound	-1.293
		Upper Bound	-3.50E-02
Quadratic	Contrast Estimate		1.149
	Hypothesized Value		0
	Difference (Estimate - Hypothesized)		1.149
	Std. Error		.312
	Sig.		.001
	95% Confidence Interval for Difference	Lower Bound	.520
		Upper Bound	1.778
Cubic	Contrast Estimate		.696
	Hypothesized Value		0
	Difference (Estimate - Hypothesized)		.696
	Std. Error		.312
	Sig.		.031
	95% Confidence Interval for Difference	Lower Bound	6.662E-02
		Upper Bound	1.325
Order 4	Contrast Estimate		-.120
	Hypothesized Value		0
	Difference (Estimate - Hypothesized)		-.120
	Std. Error		.312
	Sig.		.704
	95% Confidence Interval for Difference	Lower Bound	-.749
		Upper Bound	.510

a. Metric = 1.000, 2.000, 3.000, 4.000, 5.000

- Here are the results

$$\hat{\psi}_{linear} : t(45) = -2.13, p = .039, r = .30$$
$$\hat{\psi}_{quadratic} : t(45) = 3.68, p = .001, r = .48$$
$$\hat{\psi}_{cubic} : t(45) = 2.23, p = .031, r = .32$$
$$\hat{\psi}_{quartic} : t(45) = -0.38, p = .704, r = .06$$

  - We conclude there are significant linear, quadratic and cubic trends.

- Wait a minute . . . Didn't we just conclude there were no significant trends higher than quadratic!?!

- When the omnibus test is not significant, you still may be able to find significant contrasts. (Remember, we demonstrated that a significant contrast does not imply a significant omnibus F-test) Use combined contrast tests with caution!

ANOVA Table

Source	Sum of Squares	df	Mean Square	F	Sig.
Between Groups	22.600	4	5.650	5.792	.001
Linear Term	4.410	1	4.410	4.521	.039
Quadratic Term	13.207	1	13.207	13.538	.001
Cubic Term	4.840	1	4.840	4.961	.031
4th-order Term	.143	1	.143	.146	.704
Within Groups	43.900	45	.976		
Total	66.500	49			

- In general, to test m orthogonal contrasts simultaneously

$$F(m, dfw) = \frac{\left(\frac{SS\hat{\psi}_1 + ... + SS\hat{\psi}_m}{m}\right)}{MSW}$$

Where $m \leq a$

11. Polynomial trends with unequal cell size

- Our formulas for computing the value of a contrast, the sums of squares of a contrast, and the significance of a contrast can all handle designs with unequal *n* in each cell

$$\hat{\psi} = \sum_{j=1}^{a} c_i \bar{X}_i = c_1\bar{X}_1 + c_2\bar{X}_2 + c_3\bar{X}_3 + ... + c_a\bar{X}_a$$

$$SS\hat{\psi} = \frac{\hat{\psi}^2}{\sum \frac{c_i^2}{n_i}}$$

$$t_{observed}(N-a) = \frac{\sum c_i \bar{X}_i}{\sqrt{MSW \sum \frac{c_i^2}{n_i}}} \quad \text{or} \quad F_{observed}(1, N-a) = \frac{\hat{\psi}^2}{MSW \sum \frac{c_i^2}{n_i}}$$

- The problem is in the orthogonality of contrasts with unequal *n*

$$\psi_1 = (a_1, a_2, a_3, ..., a_a)$$
$$\psi_2 = (b_1, b_2, b_3, ..., b_a)$$

  - Two contrasts are orthogonal for unequal *n* if

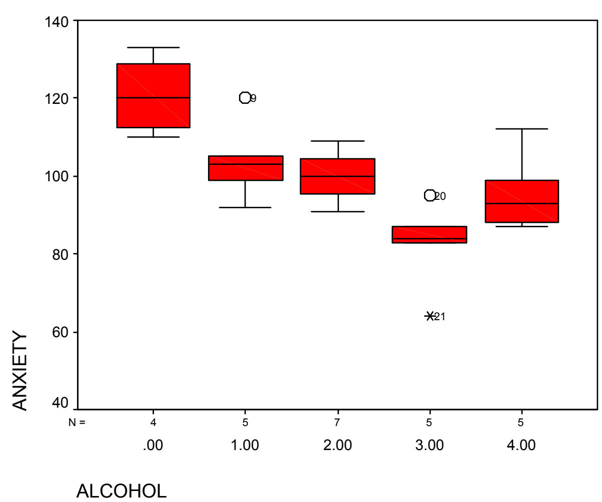
$$\sum_{j=1}^{a} \frac{a_i b_i}{n_i} = 0 \text{ or } \quad \frac{a_1 b_1}{n_1} + \frac{a_2 b_2}{n_2} + ... + \frac{a_a b_a}{n_a} = 0$$

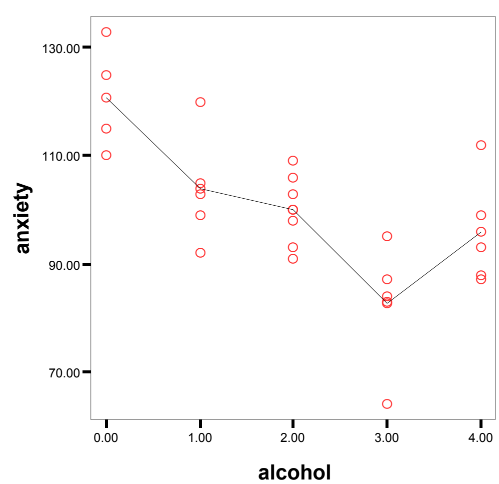
  - All of our “standard” orthogonal contrasts will no longer be orthogonal

- Example #1 with unequal *n*

  It was of interest to determine the effects of the ingestion of alcohol on anxiety level. Five groups of 50 year-old adults were administered between 0 and 4 ounces of pure alcohol per day over a one-month period. At the end of the experiment, their anxiety scores were measured with a well-known Anxiety scale.

	0oz.	1oz.	2oz.	3oz.	4oz.
	115	99	91	84	99
	133	92	103	83	93
	110	103	109	87	87
	125	105	98	95	88
		120	100	64	112
			93		
			106		
$\bar{X}_j$	120.75	103.80	100.00	82.60	95.80
n_j	4	5	7	5	5





- We would like to test for linear, quadratic, cubic and higher-order trends

```
UNIANOVA
  anxiety  BY alcohol
  /CONTRAST (alcohol)=Polynomial
  /PRINT=test(LMATRIX) .
```

Contrast Coefficients (L' Matrix)

	ALCOHOL Polynomial Contrast[a]			
Parameter	Linear	Quadratic	Cubic	Order 4
Intercept	.000	.000	.000	.000
[ALCOHOL=.00]	-.632	.535	-.316	.120
[ALCOHOL=1.00]	-.316	-.267	.632	-.478
[ALCOHOL=2.00]	.000	-.535	.000	.717
[ALCOHOL=3.00]	.316	-.267	-.632	-.478
[ALCOHOL=4.00]	.632	.535	.316	.120

The default display of this matrix is the transpose of the corresponding L matrix.

a. Metric = 1.000, 2.000, 3.000, 4.000, 5.000

- We could also use ONEWAY, but then we could not see the contrast coefficients

- SPSS generates these contrasts, let's check to see if they are orthogonal

$$\sum_{j=1}^{a} \frac{a_i b_i}{n_i} = 0$$

Linear vs. Quadratic:

$$\frac{(-.632)(.535)}{4} + \frac{(-.316)(-.267)}{5} + 0 + \frac{(.316)(-.267)}{5} + \frac{(.632)(.535)}{5} \neq 0$$

The SPSS generated contrasts are not orthogonal when ns are unequal!

- Let's see what happens when we proceed:

$$SS\hat{\psi} = \frac{\hat{\psi}^2}{\sum \frac{c_i^2}{n_i}}$$

$$\begin{aligned}\hat{\psi}_{linear} &= -.632\overline{X}_1 - .316\overline{X}_2 + 0\overline{X}_3 + .316\overline{X}_4 + .632\overline{X}_5 \\ &= -.632(120.75) - .316(103.80) + 0 + .316(82.60) + .632(95.80) \\ &= -22.484\end{aligned}$$

$$\hat{\psi}_{quad} = 12.481 \qquad \hat{\psi}_{cubic} = 5.518 \qquad \hat{\psi}_{quartic} = 8.480$$

$$SS\hat{\psi}_{linear} = \frac{(22.484)^2}{\frac{(-.632)^2}{4} + \frac{(-.316)^2}{5} + \frac{(0)^2}{7} + \frac{(.316)^2}{5} + \frac{(.632)^2}{5}} \approx 2297.82$$

$$SS\hat{\psi}_{quadratic} = \frac{(12.481)^2}{\frac{(-.632)^2}{4} + \frac{(-.316)^2}{5} + \frac{(0)^2}{7} + \frac{(.316)^2}{5} + \frac{(.632)^2}{5}} \approx 786.92$$

$$SS\hat{\psi}_{cubic} = \frac{(5.518)^2}{\frac{(-.316)^2}{4} + \frac{(.632)^2}{5} + \frac{(0)^2}{7} + \frac{(-.632)^2}{5} + \frac{(.316)^2}{5}} \approx 148.54$$

$$SS\hat{\psi}_{quartic} = \frac{(8.48)^2}{\frac{(.120)^2}{4} + \frac{(-.478)^2}{5} + \frac{(.717)^2}{7} + \frac{(-.478)^2}{5} + \frac{(.120)^2}{5}} \approx 419.74$$

ANOVA

ANXIETY

	Sum of Squares	df	Mean Square	F	Sig.
Between Groups	3395.065	4	848.766	9.171	.000
Within Groups	1943.550	21	92.550		
Total	5338.615	25			

$SS\hat{\psi}_{linear} + SS\hat{\psi}_{quadratic} + SS\hat{\psi}_{cubic} + SS\hat{\psi}_{quartic}$ $\quad SSB = 3395.07$

$= 2297.82 + 786.92 + 148.54 + 419.74$
$= 3653.02$

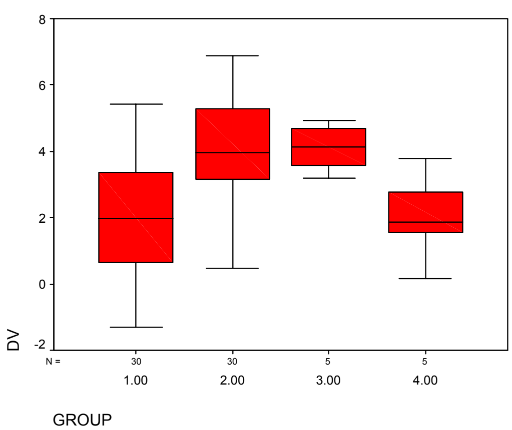
$$3653.05 \neq 3395.07$$

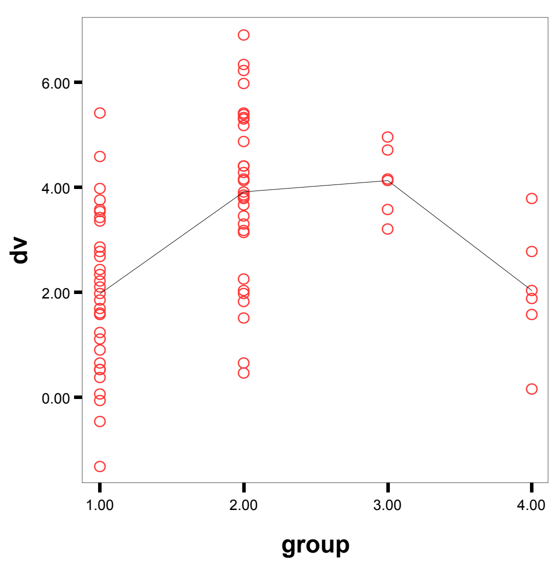
- For non-orthogonal contrasts, we can no longer decompose the sums of squares additively

- One "fix" is to weight the cell means by their cell size. Using this weighted approach is equivalent to adjusting the contrast coefficients by the cell size.

- Example #2 with unequal n

Group	1	2	3	4
$\bar{X}_j$	2	4	4	2
n_j	30	30	5	5





- Is there a linear trend in the DV?

```
ONEWAY dv BY group
 /POLY= 3.
```

ANOVA

DV

			Sum of Squares	df	Mean Square	F	Sig.
Between Groups	(Combined)		66.468	3	22.156	8.996	.000
	Linear Term	Unweighted	.062	1	.062	.025	.874
		Weighted	13.004	1	13.004	5.280	.025
	Quadratic Term	Unweighted	34.461	1	34.461	13.991	.000
		Weighted	53.306	1	53.306	21.643	.000
	Cubic Term	Unweighted	.157	1	.157	.064	.801
		Weighted	.157	1	.157	.064	.801
Within Groups			162.558	66	2.463		
Total			229.026	69			

- According to the unweighted analysis, there is no linear trend
  This analysis treats all group means equally
  The Contrast SS do not partition the *SSB*
- According to the weighted analysis, there is a linear trend
  This analysis gives most of the weight to group 1 and group 2
  The Contrast SS do partition the *SSB* exactly

- Which method is better?
  - If your goal is to compare group means, then you should conduct the unweighted analysis.
    This case holds most of the time!
    Remember, there is nothing wrong with testing non-orthogonal contrasts
    But you cannot construct combined contrasts tests

  - If the inequality in the cell sizes reflects a meaningful difference in group sizes and you want to reflect those differences in your analysis, then a weighted means approach <u>may</u> be appropriate.
    You must have a representative sample
    Your main goal would NOT be to compare groups
    If you think a weighted analysis may be appropriate, then you should read more about proper interpretation of this analysis. (see Maxwell & Delaney, 1990)

- A return to example #1: Alcohol and anxiety
  - We computed the Sums of Squares for each contrast. Let's complete our analysis

ANOVA

ANXIETY

	Sum of Squares	df	Mean Square	F	Sig.
Between Groups	3395.065	4	848.766	9.171	.000
Within Groups	1943.550	21	92.550		
Total	5338.615	25			

$$SS\hat{\psi}_{linear} = 2297.82$$
$$SS\hat{\psi}_{quadratic} = 786.92$$
$$SS\hat{\psi}_{cubic} = 148.54$$
$$SS\hat{\psi}_{quartic} = 419.74$$

$$F_{linear}(1,21) = \frac{2297.82}{92.55} = 24.83, p < .001, r = .74$$

$$F_{quad}(1,21) = \frac{786.92}{92.55} = 8.50, p = .008, r = .54$$

$$F_{cubic}(1,21) = \frac{148.54}{92.55} = 1.60, p = .22, r = .27$$

$$F_{quartic}(1,21) = \frac{419.74}{92.55} = 4.54, p = .04, r = .42$$


```
ONEWAY anxiety BY alcohol
  /POLY= 4.
```

ANOVA

ANXIETY

			Sum of Squares	df	Mean Square	F	Sig.
Between Groups	(Combined)		3395.065	4	848.766	9.171	.000
	Linear Term	Unweighted	2297.823	1	2297.823	24.828	.000
		Weighted	2144.266	1	2144.266	23.169	.000
		Deviation	1250.799	3	416.933	4.505	.014
	Quadratic Term	Unweighted	786.919	1	786.919	8.503	.008
		Weighted	677.425	1	677.425	7.320	.013
		Deviation	573.374	2	286.687	3.098	.066
	Cubic Term	Unweighted	148.538	1	148.538	1.605	.219
		Weighted	153.632	1	153.632	1.660	.212
		Deviation	419.742	1	419.742	4.535	.045
	4th-order Term	Unweighted	419.742	1	419.742	4.535	.045
		Weighted	419.742	1	419.742	4.535	.045
Within Groups			1943.550	21	92.550		
Total			5338.615	25			

- Our hand calculations exactly match the unweighted analysis

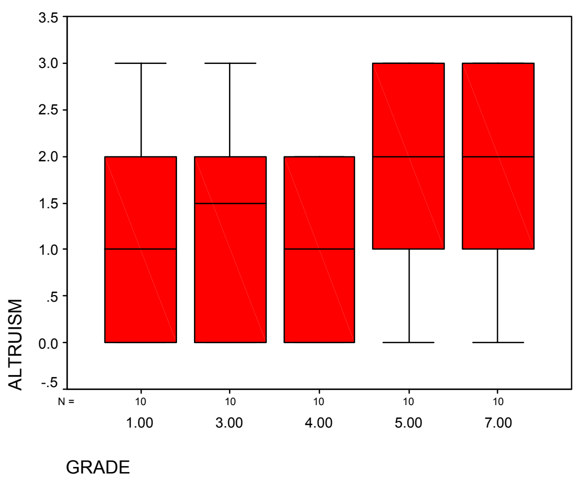
- Remember, we originally wanted to test for linear, quadratic, and all higher order terms. Because of the non-orthogonality of the contrasts, we cannot compute a deviation from linearity and quadratic trends test. We must report a test on each contrast individually.

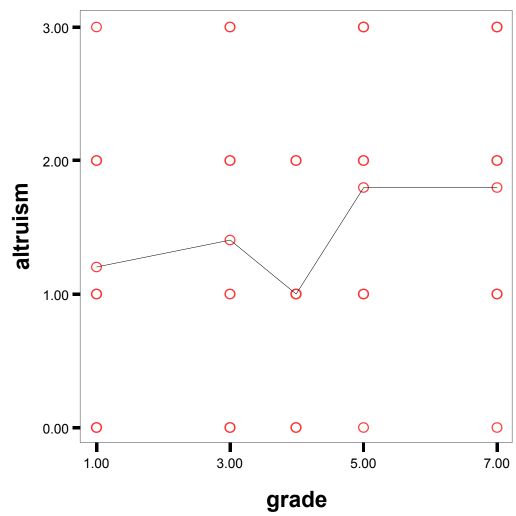
- We conclude that there is a linear, a quadratic and a 4th order effect of alcohol on anxiety
  - This 4th order effect is going to be a pain to explain in your results and discussion section!

12. A final example

- In an investigation of altruism in children, investigators examined children in 1st, 3rd, 4th, 5th, and 7th grades. The children were given a generosity scale. Below are the data collected:

Grade									
1st		*3rd*		*4th*		*5th*		*7th*	
0	1	3	2	2	1	3	0	3	2
1	3	0	1	0	2	2	3	0	1
0	2	2	3	1	1	3	1	2	2
2	2	2	0	0	2	1	1	1	3
0	1	1	0	1	0	2	2	3	1





  - We want to investigate if there is a linear increase in altruism
    - The cell sizes are equal
    - But the spacing is not

- We cannot use the tabled values for trend contrasts. We must let SPSS compute them for us

```
UNIANOVA altruism BY grade
  /CONTRAST (grade)=Polynomial (1,3,4,5,7)
  /PRINT = test(lmatrix).
```

Tests of Between-Subjects Effects

Dependent Variable: ALTRUISM

Source	Type III Sum of Squares	df	Mean Square	F	Sig.
Corrected Model	5.120[a]	4	1.280	1.220	.316
Intercept	103.680	1	103.680	98.847	.000
GRADE	5.120	4	1.280	1.220	.316
Error	47.200	45	1.049		
Total	156.000	50			
Corrected Total	52.320	49			

a. R Squared = .098 (Adjusted R Squared = .018)

Contrast Coefficients (L' Matrix)

	GRADE Polynomial Contrast[a]			
Parameter	Linear	Quadratic	Cubic	Order 4
Intercept	.000	.000	.000	.000
[GRADE=1.00]	-.671	.546	-.224	4.880E-02
[GRADE=3.00]	-.224	-.327	.671	-.439
[GRADE=4.00]	.000	-.436	.000	.781
[GRADE=5.00]	.224	-.327	-.671	-.439
[GRADE=7.00]	.671	.546	.224	4.880E-02

The default display of this matrix is the transpose of the corresponding L matrix.

a. Metric = 1.000, 3.000, 4.000, 5.000, 7.000

Contrast Results (K Matrix)

GRADE Polynomial Contrast[a]			Dependent Variable ALTRUISM
Linear	Contrast Estimate		.492
	Hypothesized Value		0
	Difference (Estimate - Hypothesized)		.492
	Std. Error		.324
	Sig.		.136
	95% Confidence Interval for Difference	Lower Bound	-.160
		Upper Bound	1.144
Quadratic	Contrast Estimate		.153
	Hypothesized Value		0
	Difference (Estimate - Hypothesized)		.153
	Std. Error		.324
	Sig.		.639
	95% Confidence Interval for Difference	Lower Bound	-.500
		Upper Bound	.805
Cubic	Contrast Estimate		-.134
	Hypothesized Value		0
	Difference (Estimate - Hypothesized)		-.134
	Std. Error		.324
	Sig.		.681
	95% Confidence Interval for Difference	Lower Bound	-.786
		Upper Bound	.518
Order 4	Contrast Estimate		-.478
	Hypothesized Value		0
	Difference (Estimate - Hypothesized)		-.478
	Std. Error		.324
	Sig.		.147
	95% Confidence Interval for Difference	Lower Bound	-1.130
		Upper Bound	.174

a. Metric = 1.000, 3.000, 4.000, 5.000, 7.000

- We find no evidence for any trends in the data,
  all F's$(1, 45) < 2.31$, p's $> .13$

- The purpose of the study was to examine linear trends, so can I test the linear trend and the deviation from a linear trend?

  - Can I use the ONEWAY command to do so?


```
ONEWAY altruism BY grade
 /POLYNOMIAL= 1.
```

ANOVA

ALTRUISM

			Sum of Squares	df	Mean Square	F	Sig.
Between Groups	(Combined)		5.120	4	1.280	1.220	.316
	Linear Term	Contrast	2.420	1	2.420	2.307	.136
		Deviation	2.700	3	.900	.858	.470
Within Groups			47.200	45	1.049		
Total			52.320	49			

- Yes, but only if we have grade coded as (1, 3, 4, 5, 7).
  Remember, ONEWAY uses the spacing provided in your coding of the data

- We can report no evidence of a linear trend, $F(1,45) = 2.31, p = .14, \omega^2 = .03$, and no evidence for any higher order trends, $F(3,45) = 0.86, p = .47, \omega^2 < .01$