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# Integer de Casteljau Algorithm for Rasterizing NURBS Curves

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**Abstract**

*An integer version of the well-known de Casteljau algorithm of NURBS curves is presented here. The algorithm is used to render NURBS curves of any degree on a raster device by turning on pixels that are closest to the curve. The approximation is independent of the parametrization, that is, it is independent of the weights used. The algorithm works entirely in the screen coordinate system and produces smooth rendering of curves without oversampling. Because of the integer arithmetic used, the algorithm is easily cast in hardware.*

## 1. Introduction

Non-uniform rational B-spline curves, commonly referred to as NURBS, have become standard tools for representing many geometric entities used in a variety of visualization oriented fields such as Computer Graphics, CADCAM and Vison/Image Processing[1,2]. While the computation of Points along a NURBS curve has been well established, there seems to be no generally accepted technique for the fast and accurate display of them. Some of the existing integer algorithms known to the graphics community are capable of rendering such simple entities as lines, circles and a few conic types such as the ellipse in the standard position (the major and the minor axes are parallel to the $x$ and $y$ axes of the coordinate system). In this paper we look at the problem of integerizing the well-known de Casteljau algorithm to obtain a smooth rendering of NURBS curves on a raster device. Because of the integer arithmetic used, and because of the simplicity of the algorithm, the technique is an excellent candidate for hardware implementation.

First, we review some definitions of NURBS curves to enable the reader to follow the ideas of the paper. A short review of different drawing techniques of NURBS curves is followed by a section on the de Casteljau algorithm that is the prime candidate for integerization. Decomposition of NURBS curves into piecewise Bezier curves is outlined in the subsequent section. In the last two sections we elaborate on the details of how non-rational and rational curves are rendered in integer arithmetic using the idea inherent in the de Casteljau algorithm.

## 2. NURBS: Definition and Basic Properties

A NURBS curve of degree $p$ is a vector-valued piecewise rational polynomial function of the form[3-5,7]:

$$C(u) = \frac{\sum_{i=0}^{n} w_i P_i N_{i,p}(u)}{\sum_{i=0}^{n} w_i N_{i,p}(u)}$$

where the vectors $P_i$ denote the so-called *control polygon*, the $w_i$ denote the *weights* associated with each control point, and $N_{i,p}(u)$ are the degree $P$ non-periodic B-splines defined over the *knot vector*

$$U = \{\underbrace{0,0,\ldots,0}_{p+1}, u_{p+1}, \ldots, u_{m-p-1}, \underbrace{1,1,\ldots,1}_{p+1}\}$$

where $m = n+p+1$. For ease of understanding and computation, the following *rational basis functions* are introduced:

$$R_{i,p}(u) = \frac{w_i N_{i,p}(u)}{\sum_{j=0}^{n} w_j N_{j,p}(u)}$$