

Factorial design

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{The term refers to experiments where two or more factors are varied independently. We mostly focus on the case of two factors, which exhibits most of the techniques and difficulties.

{In many situations, the most important question in factorial design is that of interaction between factors. In the absence of interactions, the effect of varying one factor has only the effect of simultaneously shifting the distributions of treatment groups determined by the other factor. When the factors are interacting, the effect may be more complex.

{The factors may be qualitative or quantitative. When one or more factors are quantitative, typically one wants to perform regression on these factors. Orthogonal polynomials may be used as a preferred way to perform regression, both in the case of a single quantitative factor, as well as more than one quantitative factors. The principles of calculations with multi-dimensional orthogonal polynomials remain the same as in the case of one quantitative factor, i.e. they are obtained by Gram-Schmidt process, and estimators of regression coefficients are given by Bessel formulas.

{A two-way analysis of variance is the analysis of variance of a factorial design with two factors. The full two-dimensional model with two qualitative factors is given by these equations:

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$$y_{ijk} = \mu + \alpha_i + \beta_j + \gamma_{ij} + \epsilon_{ijk}$$

{where $i = 1, 2, \dots, a$, $j = 1, 2, \dots, b$ and $k = 1, 2, \dots, r_{ij}$. Theoretically important case is that of $r_{ij} = r$ being the same for all cells (groups with the same factor levels given by indices i and j).

{The term γ_{ij} accounts for the interactions between factors. In the absence of interactions we have a reduced model:

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$$y_{ijk} = \mu + \alpha_i + \alpha_j + \epsilon_{ijk}$$

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