

The General Linear Model

<http://marekrychlik.com/node/30>

The general linear model (GLM) captures the relationship between one or more response variables y , and several design variables x_1, x_2, \dots, x_k . One postulates a relationship between y and x_i which involves parameters $\beta_j, j = 1, 2, \dots, k$:

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$$y = \beta_0 + \sum_{j=1}^k \beta_j x_j + e$$

where e represents the error. Typically, we assume that e has mean 0, i.e. that the error is unbiased.

Variables x_j could be interval or ratio measurements (such as mass, time, temperature etc.), or can be indicator variables. An indicator variable has value of 0 or 1. A set of indicator variables may be assigned to indicate the membership of a unit to a particular treatment group. Thus, if there are t treatment groups, we would have t indicator variables. The i -th indicator variable of the set would have value of 1 for all units in group i and 0 otherwise.

In the meat packaging example there are 4 treatment labels, and thus there are 4 indicator variables x_1-x_4 . The GLM for this example would be (assuming there is no β_0):

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$$y = \sum_{i=1}^4 \beta_i x_i + e.$$

If we restrict the model to the i -th treatment group, we can write:

$$y_{ij} = \beta_i + \epsilon_{ij}$$

which is equivalent to the full model introduced before (formally setting $\mu_i = \beta_i$). Similarly, we can write the reduced model as a GLM:

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$$y = \beta_0 + e.$$

The parameters of the GLM are estimated using the least squares method.